CALCULUS II – Worksheet #16

1. Find all absolute and relative maximums and minimums for $y = 2x^3 - 3x^2 - 12x$ on $[-2,3]$.

2. Find all critical points, inflection points, and sketch $y = x^4 - 4x^3$.

3. Find all intervals where $y = x^4 - 4x^3 + 4x^2$ is increasing and decreasing.

4. $f(x) = x^4 - 4x^2$ has: (A) 1 max, 2 min (B) 1 min, 2 max (C) 1 min, 1 max (D) 2 max, 0 min (E) 2 min, 0 max

5. The number of inflection points of the curve in problem 4 is:
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

6. The total number of relative maximum and minimum points of the function whose derivative is $f'(x) = x^2(x+1)^3(x-4)^3$ is (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

7. The sum of one number and twice another is 24. Find the two numbers so that their product is a maximum.

8. A square piece of tin has 12 inches on a side. An open box is formed by cutting out equal square pieces at the corners and bending upward the projecting portions which remain. Find the maximum volume that can be obtained.

9. Find all absolute and relative maximum and minimum points for $y = x^3 - 3x + 1$ on $[-2,3]$.

10. The base of a solid is the region enclosed by the graph of $x = 1 - y^2$ and the y-axis. If all plane cross-sections perpendicular to the x-axis are semicircles with diameters parallel to the y-axis, then the volume is:
A) $\frac{\pi}{8}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{2}$ D) $\frac{3\pi}{4}$ E) $\frac{3\pi}{2}$

ANSWERS:

1. **(–1,7) abs and rel max, (2,–20) abs and rel min**
2. **(0,0) terrace, (3,–27) rel min, (0,0) and (2,–16) inf pts**
3. **inc: $0 < x < 1$ and $x > 2$ dec: $x < 0$ and $1 < x < 2$**
4. **A**
5. **C**
6. **C**
7. **12 and 6**
8. **128**
9. **(1,–1) rel and abs min, (–1,3) rel max, (–2,–1) abs min, (3,19) abs max**
10. **B**

CALCULUS II – Worksheet #14

For problems 1– 3, find the intervals where the curve is increasing and decreasing, find and label all critical points (tell whether they are rel max, rel min, or terrace points), **<u>and draw the curve on graph paper</u>**. Make sure to show a first derivative chart.

1. $f(x) = 3x^2 - 3x + 2$

2. $f(x) = x^3 - x^2 - x$

3. $f(x) = 2x^3 - 9x^2 + 2$

4. The function $f(x) = x^3 - 2x^2$ is increasing on which of the following interval(s)?
   A) $x < 0$ only B) $x > \frac{4}{3}$ only C) $0 < x < \frac{4}{3}$ •D) $x < 0$ or $x > \frac{4}{3}$ E) $x < 0$ or $x > \frac{3}{4}$

5. Given $f(x) = \frac{1}{5}x^5 - \frac{1}{24}x^4$, find where the relative extrema (critical points) of $f'''(x)$ occur.
   A) $x = 0, \frac{1}{12}$ B) $x = \frac{1}{8}$ •C) $x = \frac{1}{24}$ D) $x = 0, \frac{1}{8}$ E) $x = 0, \frac{1}{6}$

6. If $g(x) = \frac{1}{2}|3 - x|$, then the value of the derivative of $g(x)$ at $x = 3$ is
   A) $-\frac{1}{2}$ B) $\frac{1}{2}$ C) 0 D) 3 •E) nonexistent

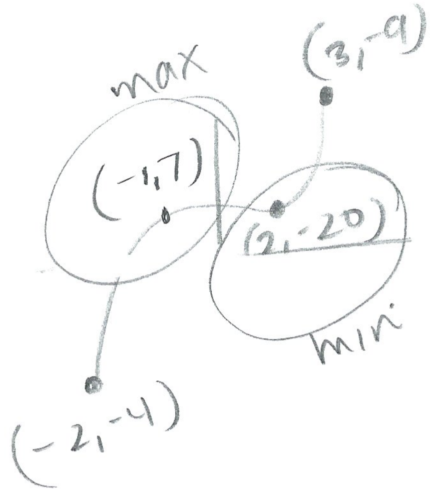
7. ~~The average value of the function $f(x) = x \sin x$ on $[1,\pi]$ is approximately~~
   ~~•A) 1.326 B) 1.467 C) 2.840 D) 3.142 E) 4.076~~

ANSWERS – Worksheet #14

1. $\left(\frac{1}{2}, \frac{5}{4}\right)$ **rel min, inc** $x > \frac{1}{2}$, **dec** $x < \frac{1}{2}$
2. **(1,–1) rel min,** $\left(\frac{-1}{3}, \frac{5}{27}\right)$ **rel max, inc x < –1/3 or x > 1, dec –1/3 < x < 1**
3. **(0,2) rel max, (3, –25) rel min, inc x < 0 or x > 3, dec 0 < x < 3**
4. **D**
5. **C**
6. **E**
7. **A**

Worksheet #16

① $y = 2x^3 - 3x^2 - 12x$ $\quad [-2, 3)$



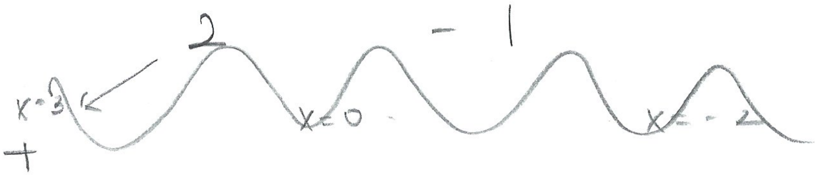
$y' = 6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$ $\quad$ -2, -2, 1, -1

$6(x-2)(x+1) = 0$

$x = 2, -1$

slope $= 0$ at $x = 2, -1$



-1 $\qquad$ 2

x=-2 $\qquad$ x=0 $\qquad$ x=3

+ $\qquad$ − $\qquad$ +

max at $x = -1$ $\qquad$ min at $x = 2$

$x = -1$

$y = 2(-1)^3 - 3(-1)^2 - 12(-1)$

$-2 - 3 + 12$

$y = 7$

$x = 2$

$y = 2(2)^3 - 3(2)^2 - 12(2)$

$= 16 - 12 - 24$

$= 4 - 24 = -20$

rel. max: abs max $(-1, 7)$

rel min abs. min $(2, -20)$

check endpoints

$f(-2) = 2(-2)^3 - 3(-2)^2 - 12(-2)$

$= -16 - 12 + 24$

$= -28 + 24$

$= -4$

$(-2, -4)$

$f(3) = 2 \cdot 3^3 - 3 \cdot 3^2 - 12 \cdot 3$

$= 54 - 27$

$27 - 36$

-9

$(3, -9)$

② $y = x^4 - 4x^3$

critical points $y' = 4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

$x = 0 \quad x = 3$

$x = 0 \quad y = 0^4 - 4(0)^3$

$y = 0$

$$\begin{array}{r} 27 \\ \times 4 \\ \hline 108 \end{array}$$

$x = 3 \quad y = 3^4 - 4(3)^3$

$y = 81 - 108 = -27$

critical points
(0,0)
(3,-27)

$x = 0 \qquad x = 3$

$x = -1 \qquad x = 1 \qquad x = 4$

$- \qquad -$ min $\qquad +$

Inflection point

$y'' = 12x^2 - 24x = 0$

$12x(x-2) = 0$

$x = 0 \quad x = 2$

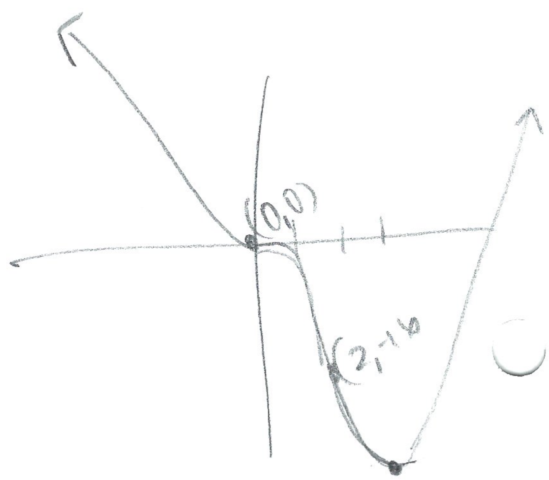
Inflection points
(0,0)
(2,-16)

$x = 2$

$y = 2^4 - 4(2)^3$

$= 16 - 4(8)$

$16 - 32 = -16$



(3) $y = x^4 - 4x^3 + 4x^2$

increasing / decreasing

$$y' = 4x^3 - 12x^2 + 8x$$

$$= 4x(x^2 - 3x + 2) = 0$$

$$= 4x(x-2)(x-1) = 0$$

$x = 0 \quad x = 2 \quad x = 1$

	0	1	2
$x = -1$	$x = \frac{1}{2}$	$x = 1\frac{1}{2}$	$x = 3$
$-$	$+$	$-$	$+$

increasing	$0 < x < 1$	$x > 2$
decreasing	$x < 0$	$1 < x < 2$

(4) $f(x) = x^4 - 4x^2$

$y' = 4x^3 - 8x = 0$

$4x(x^2 - 2) = 0$

$x = 0 \quad x = \pm\sqrt{2}$

$x = -\sqrt{2} \quad x = 0 \quad x = \sqrt{2}$

$x = -4 \quad x = -1 \quad x = 1 \quad x = 4$

$- \quad + \quad - \quad +$

min max min

$\boxed{\begin{array}{c}\text{2 mins} \\ \text{1 max}\end{array}}$

A

(15) $f(x) = x^4 - 4x^2$

$f' = 4x^3 - 8x$

$f'' = 12x^2 - 8 = 0$

$= 4(3x^2 - 2) = 0$

$= 3x^2 - 2 = 0$

$\frac{\cancel{3}x^2}{\cancel{3}} = \frac{2}{3}$

$\sqrt{x^2} = \sqrt{\frac{2}{3}}$ $x = \pm\sqrt{\frac{2}{3}}$

$\boxed{\text{2 inflection points}}$ C

(16) $f'(x) = x^2(x+1)^3(x-4)^3 = 0$

$x = 0$ $x = -1$ $x = 4$

$x = -1$ $x = 0$ $x = 4$

$x = -2$ $x = -\frac{1}{2}$ $x = 1$ $x = 5$

$+ - \cdot -$ $+ \cdot - -$ $+ + -$

$+$ $+$ max $-$ min $+$ $= \boxed{2}$

C

⑨ $y = x^3 - 3x + 1$ $[-2, 3]$

$y' = 3x^2 - 3$

$= 3(x^2 - 1)$

$= 3(x+1)(x-1) = 0$

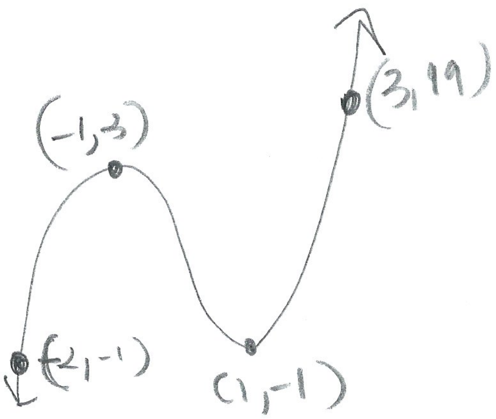
$x = 1, -1$

$x = -1$ $x = 1$

$x = -2$ $x = 0$ $x = 2$

$+$ $-$ $+$

max min



relative max $= (-1, 3)$

abs. max $(3, 19)$

relative min / abs min $(1, -1)$

abs min $(-2, -1)$

$x = -1$, $y = (-1)^3 - 3(-1) + 1$

$= -1 + 3 + 1$

$= 2 + 1 = 3$

check endpoints

$x = -2$ $y = (-2)^3 - 3(-2) + 1$

$-8 + 6 + 1$

$-2 + 1 = -1$

$x = 3$

$3^3 - 3(3) + 1$

$27 - 9 + 1$

$19 + 1 = 19$

$1^3 - 3(1) + 1$

$1 - 3 + 1$

-1

Worksheet #14

① $f(x) = 3x^2 - 3x + 2$

$f'(x) = 6x - 3 = 0$

$\frac{\cancel{6}x}{\cancel{6}} = \frac{3}{6}$

$x = \frac{1}{2}$ critical point

min

$x = \frac{1}{2}$

$x = 0$ $x = 1$

$-$ $+$

$f''(x) = 6$ positive #
concave up for all x

increase $x > \frac{1}{2}$

decrease $x < \frac{1}{2}$

minimum at $x = \frac{1}{2}$

$y = 3\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 2$

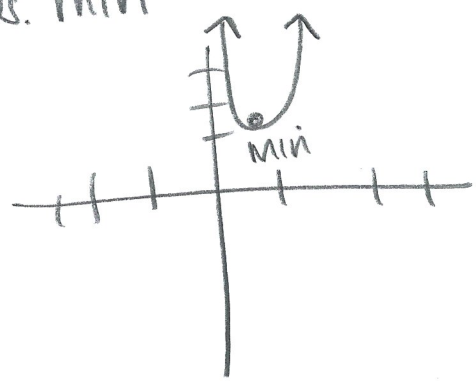
$= \frac{3}{4} - \frac{3 \cdot 2}{2 \cdot 2} + \frac{2 \cdot 4}{1 \cdot 4}$

$= \frac{3}{4} - \frac{6}{4} + \frac{8}{4} =$

$-\frac{3}{4} + \frac{8}{4} = \frac{5}{4}$

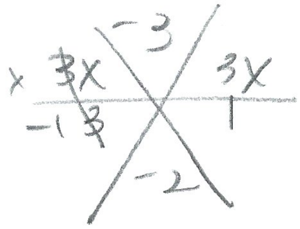
min. at $\left(\frac{1}{2}, \frac{5}{4}\right)$

abs. min



② $f(x) = x^3 - x^2 - x$

$f'(x) = 3x^2 - 2x - 1 = 0$



$(x-1)(3x+1) = 0$

$\boxed{x = 1, \frac{-1}{3}}$

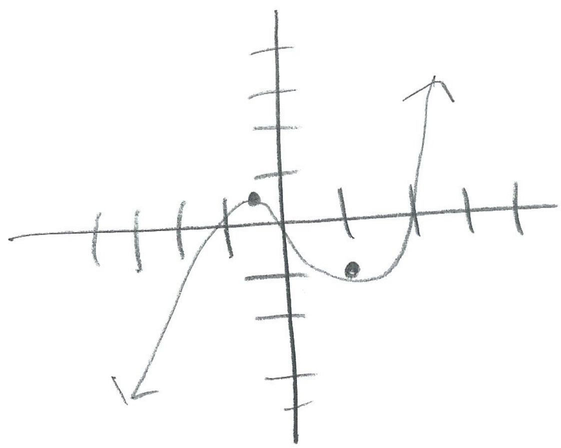
critical points

$x = \frac{-1}{3}$ $\quad$ $x = 1$

$x = 1$ $\quad$ $x = \frac{2}{3}$ $\quad$ $x = 2$

$+$ $\quad$ max $\quad$ $-$ $\quad$ min $\quad$ $+$

$$\boxed{\begin{array}{l} \text{max} = \left(\frac{-1}{3}, \frac{5}{27}\right) \\ \text{min} = (1, -1) \end{array}}$$



$f''(x) = 6x - 2 = 0$

$\frac{6x}{6} = \frac{2}{6}$

$x = \frac{1}{3}$

$$\boxed{\begin{array}{ll} \text{increase} & x < \frac{-1}{3} \quad x > 1 \\ \text{decrease} & -\frac{1}{3} < x < 1 \end{array}}$$

(3) $f(x) = 2x^3 - 9x^2 + 2$

$f'(x) = 6x^2 - 18x = 0$

$6x(x-3) = 0$

$\boxed{x=0 \quad x=3}$ critical points

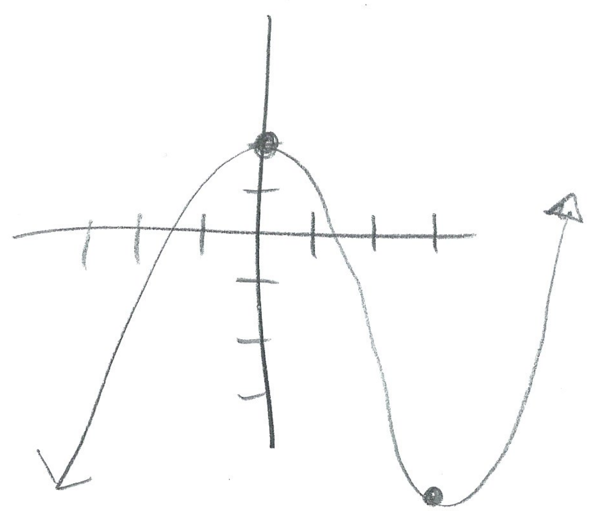
$x=0$ $\qquad$ $x=3$

$x=-1$ $\qquad$ $x=1$ $\qquad$ $x=4$

$+$ max $-$ min $+$

max = $(0, 2)$ plugin $x=0$

min = $(3, -25)$ plugin $x=3$



Increase $x > 3$ $\quad$ $x < 0$

Decrease $0 < x < 3$

(6) $g(x) = \frac{1}{2}|3-x|$ at $x=3$

$g'(x) = \frac{1}{2|3-x|} \cdot -1$

(4) $f(x) = x^3 - 2x^2$

$f'(x) = 3x^2 - 4x$

$= x(3x-4) = 0$

$x = 0 \quad x = \frac{4}{3}$

$x=-1 \qquad x=1 \qquad x=2$

$+ \qquad - \qquad +$

increase $\boxed{x<0 \text{ and } x>\frac{4}{3}}$ D

(5) $f(x) = \frac{1}{5}x^5 - \frac{1}{24}x^4$

$f'(x) = \frac{1}{5} \cdot 5x^4 - \frac{1}{24} \cdot 4x^3$

$f'(x) = x^4 - \frac{1}{6}x^3$

$f''(x) = 4x^3 - \frac{1}{6} \cdot 3x^2$

$\cancel{x(12x-1)=0}$

$f''(x) = 4x^3 - \frac{1}{2}x^2$

$\cancel{x=0 \quad x=\frac{1}{12}}$

$f'''(x) = 12x^2 - \frac{1}{2} \cdot 2x$

critical points

$f''''(x) = 12x^2 - x$

$f'''''(x) = 24x - 1 = 0$

$\boxed{x = \frac{1}{24}}$

C