nth root test two

(35) $\sum_{n=1}^{\infty} \frac{1}{5^n} \Rightarrow \lim_{n\to\infty} \sqrt[n]{\frac{1^n}{5^n}} = \lim_{n\to\infty} \sqrt[n]{\left(\frac{1}{5}\right)^n} = \frac{1}{5}^{n/n} = \frac{1}{5}$

$= \lim_{n\to\infty} \frac{1}{5} = \frac{1}{5} < 1$

$\boxed{\text{converges}}$

(36) $\sum_{n=1}^{\infty} \frac{1}{n^n} \Rightarrow \lim_{n\to\infty} \sqrt[n]{\frac{1}{n^n}} = \sqrt[n]{\left(\frac{1}{n}\right)^n} = \lim_{n\to\infty} \left(\frac{1}{n}\right)^{n/n}$

$= \lim_{n\to\infty} \frac{1}{n} = 0 < 1$

$\boxed{\text{converges}}$

(37) $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n \Rightarrow \lim_{n\to\infty} \sqrt[n]{\left(\frac{n}{2n+1}\right)^n} = \lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2} < 1$

$\boxed{\text{converges}}$

(38) $\sum_{n=1}^{\infty} \left(\frac{2n}{n+1}\right)^n \Rightarrow \lim_{n\to\infty} \sqrt[n]{\left(\frac{2n}{n+1}\right)^n} = \lim_{n\to\infty} \frac{2n}{n+1} = 2 > 1$

$\boxed{\text{diverges}}$

(39) $\sum_{n=1}^{\infty} \left(\frac{3n+2}{n+3}\right)^n \Rightarrow \lim_{n\to\infty} \sqrt[n]{\left(\frac{3n+2}{n+3}\right)^n} = \lim_{n\to\infty} \frac{3n+2}{n+3} = 3 > 1$

$\boxed{\text{diverges}}$

(40) $\sum_{n=1}^{\infty} \left(\frac{n-2}{5n+1}\right)^n \Rightarrow \lim_{n\to\infty} \sqrt[n]{\left(\frac{n-2}{5n+1}\right)^n} = \lim_{n\to\infty} \frac{n-2}{5n+1} = \frac{1}{5} < 1$

$\boxed{\text{converges}}$

(41) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^n} \Rightarrow \lim_{n\to\infty} \sqrt[n]{\left(\frac{-1}{\ln n}\right)^n} = \lim_{n\to\infty} \frac{-1}{\ln n} = 0 < 1$

$\boxed{\text{converges}}$

(42) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{-3n}{2n+1}\right)^{3n} \Rightarrow \lim_{n\to\infty} \sqrt[n]{\left(\frac{3n}{2n+1}\right)^{3n}} = \lim_{n\to\infty} \left(\frac{-3n}{2n+1}\right)^3$

$\displaystyle = \lim_{n\to\infty} \frac{-27n^3}{8n^3+\dots} = -\frac{27}{8} < 1$

$\boxed{\text{converges}}$

(43) $\displaystyle\sum_{n=1}^{\infty} (2\sqrt[n]{n}+1)^n \Rightarrow \lim_{n\to\infty} \sqrt[n]{(2\sqrt[n]{n}+1)^n} = \lim_{n\to\infty} 2\overset{1}{\sqrt[n]{n}}+1$

$= 2\cdot 1+1 = 3 > 1$

$\boxed{\text{diverges}}$

(44) $\displaystyle\sum_{n=1}^{\infty} e^{-3n} \Rightarrow \lim_{n\to\infty} \sqrt[n]{e^{-3n}} = e^{-3n/n} = \lim_{n\to\infty} e^{-3} = \frac{1}{e^3} < 1$

$\boxed{\text{converge}}$

(45) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3^n} \Rightarrow \lim_{n\to\infty} \sqrt[n]{\frac{n}{3^n}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}}{\sqrt[n]{3^n}} = \lim_{n\to\infty} \frac{1}{3} = \frac{1}{3} < 1$

$\boxed{\text{converge}}$

(40) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{500}\right)^n \lim_{n\to\infty} \sqrt[n]{\left(\frac{n}{500}\right)^n} = \lim_{n\to\infty} \frac{n}{500} = \infty > 1$ $\boxed{\text{diverges}}$

(48) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\ln n}{n}\right)^n \Rightarrow \lim_{n\to\infty} \sqrt[n]{\left(\frac{\ln n}{n}\right)^n} = \lim_{n\to\infty} \frac{\ln n}{n} = 0 < 1$

$\boxed{\text{diverges}}$

$\displaystyle\lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$