Calc 1 Worksheet #41
Approximating Areas using Reimann Sums

1 Approximate the area under $y=(x-1)^2$ on [0,4] using
(a) 4 rectangles whose height is given using the left endpoint
(b) 4 rectangles whose height is given using the right endpoint
(c) 4 rectangles whose height is given using the midpoint
(d) 4 trapezoids.
(e) Evaluate the integral directly.

2 Approximate the area under $y = x^2 - 1$ on [0,4] using
(a) 4 rectangles whose height is given using the left endpoint
(b) 4 rectangles whose height is given using the right endpoint
(c) 4 rectangles whose height is given using the midpoint
(d) 4 trapezoids
(e) (e) Evaluate the integral directly.

3 Approximate to 3 decimal places the integral $\int_0^4 \sqrt{x}$ with 4 equal intervals using:

a) rectangles whose height is the right-hand endpoint
b) rectangles whose height is the left-hand endpoint
c) rectangles whose height is the midpoint of the interval
d) trapezoids (trapezoidal rule)
e) Evaluate the integral directly.

4 Approximate the area under $y = (x + 1)^2$ on [0, 4] using
(a) 4 rectangles whose height is given using the left endpoint,
(b) 4 rectangles whose height is given using the right endpoint,
(c) 4 rectangles whose height is given using the midpoint, and
(d) 4 trapezoids.
(e) Evaluate the integral directly.

5 If a chart of values for f(x) =

x	-3	0	3	6	9	12	15
F(x)	-1	0	1	3	1	0	-1

Find a trapezoidal approximation of $\int_{-3}^{15} f(t)dt$ using six subintervals of length $\Delta t = 3$

6 If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

7 If $f(x) = \begin{cases} 2x & \text{for } x \le 1 \\ 3x^2-1 & \text{for } x > 1 \end{cases}$ then find $\int_0^2 f(x)dx$.

8 If $V = \frac{4}{3}\pi r^3$, what is $\frac{dV}{dr}$ when $r = 3$?

9 If $f(x) = x \cos\frac{1}{x}$, then $f'\left(\frac{2}{\pi}\right) =$

10 $\lim_{x\to 4} \dfrac{x^3-4x^2-x+4}{x-4}$

11 The solution set of $\dfrac{7}{x^2+8x+23} = 1$ is

12 Why does $f(x) = \dfrac{x^2-4x}{x-2}$ on [0,4] <u>not</u> satisfy the hypotheses of Rolle's Theorem?

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13	Find c for the Mean Value Theorem if $f(x) = 2x^2 + 1$ in [1,3].
14	A function f that is continuous for all real numbers x has $f(3) = -1$ and $f(7) = 1$. If $f(x) = 0$ for exactly one value of x, then which of the following could be x? A) −1 B) 0 C) 1 D) 4 E) 9

Answers:

1 a) 6 b) 14 c) 9 d) 10 e) 28/3	2 a) 12 b) 26 c) 37/2 d) 19 e) ~~56/3~~ $\frac{52}{3}$	3 a) 6.146 b 4.146 c) 5.384 d) 5.146 e) 5.333	4 a) 30 b) 54 c) 41 d) 42 e) $\frac{124}{3}$	5 12
6 Not defined	7 7	8 36π	9 $\frac{\pi}{2}$	10 15
11 $\{-4, -4\}$	12 f(2) DNE, therefore not continuous and f' (2) is undefined	13 2	14 4	

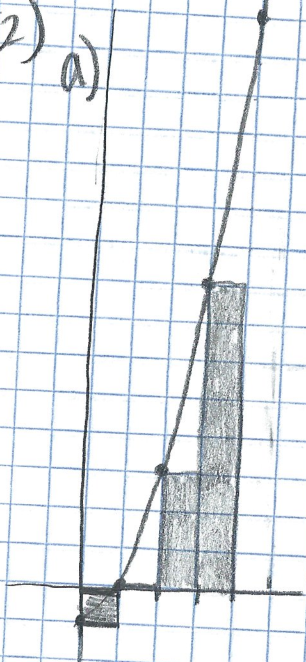
#1) a)

b)

[redacted]

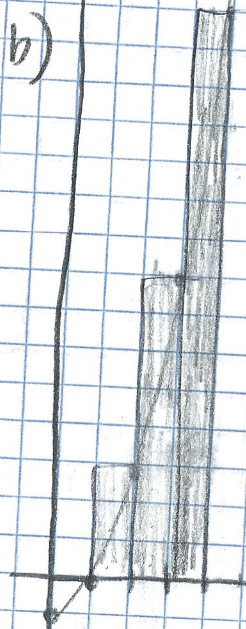
#2)

a)



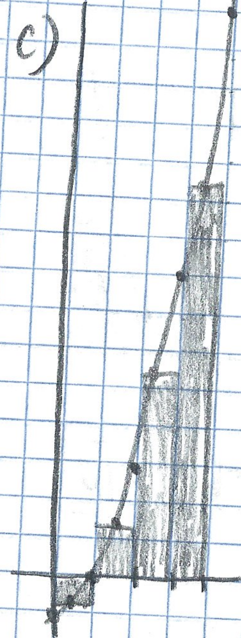
left = $\boxed{12}$

b)



right = $\boxed{26}$

c)

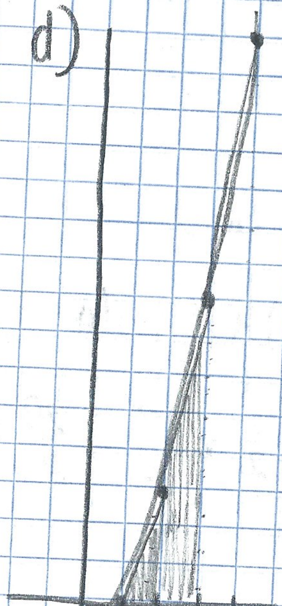


midpoint =

$1(.75) + 1(1.25) + 1(5.25) + 1(11.25)$

$18.5 = 18\frac{1}{2} = \boxed{\frac{37}{2}}$

d)



e) $\int_0^4 x^2 - 1$

$= \frac{x^3}{3} - x \Big|_0^4$

$\frac{4^3}{3} - 4 - \left(\frac{0^3}{3} - 0\right)$

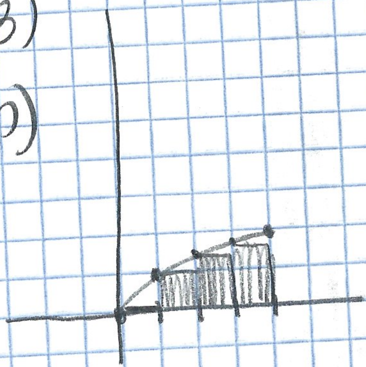
$\frac{64}{3} - \frac{4}{1} \cdot \frac{3}{3} = \frac{64}{3} - \frac{12}{3} = \boxed{\frac{52}{3}}$

$\frac{1}{2}(1)(1) + \frac{1}{2}(1)(3) + \frac{1}{2}(1)(3+8) + \frac{1}{2}(8+15)$

$\frac{1}{2} + \frac{3}{2} + \frac{11}{2} + \frac{23}{2} = \frac{38}{2} = \boxed{19}$

#3)

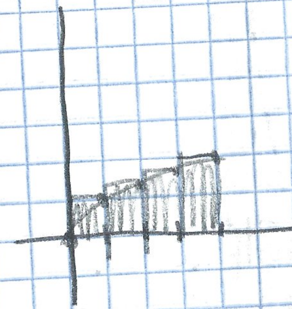
b)



left $= 1\cdot1 + 1\cdot(1.414) + 1\cdot(1.732)$

$= \boxed{4.146}$

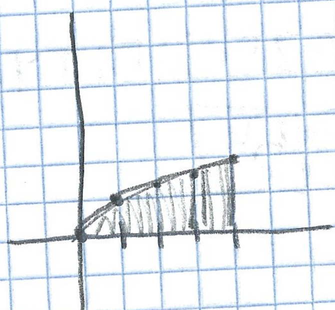
a)



right $= 1(1) + 1(1.414) + 1(1.732)$
$+ 1(2)$

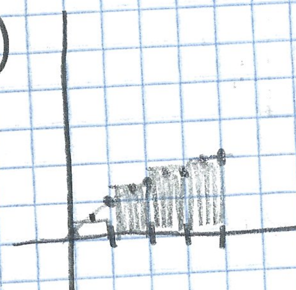
$= \boxed{6.146}$

d)



Trapezoid $= \frac{1}{2}(1)(1) + \frac{1}{2}(1)(1+1.414)$
$+ \frac{1}{2}(1)(1.414+1.732)$
$+ \frac{1}{2}(1)(1.732+2)$
$= \boxed{5.146}$

c)



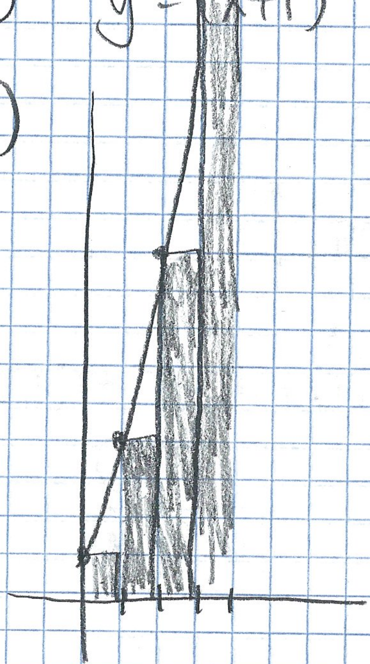
midpoint =
$1(.707) + 1(1.225)$
$+ 1(1.581) + 1(1.871)$
$= \boxed{5.384}$

$$\int_0^4 \sqrt{x}\,dx = \int_0^4 x^{1/2}\,dx = \frac{2x^{3/2}}{3}\Big|_0^4$$

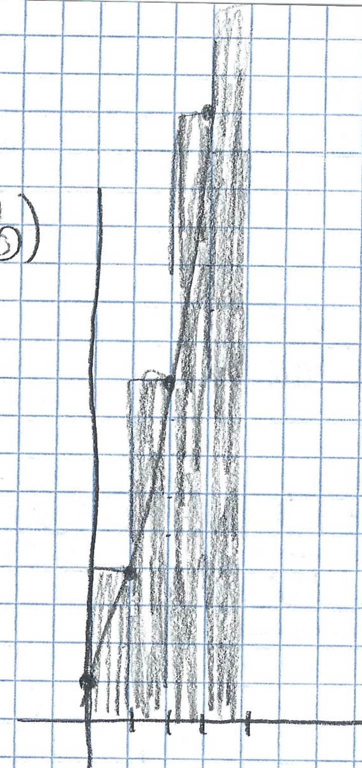
$$\frac{2}{3}\cdot 4^{3/2} = \frac{2}{3}\cdot 8 = \frac{16}{3} = \boxed{5.333}$$

#4) $y = (x+1)^2 \quad [0,4]$

a)



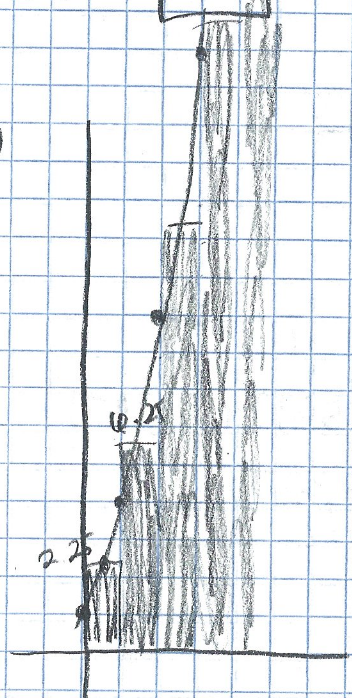
b)



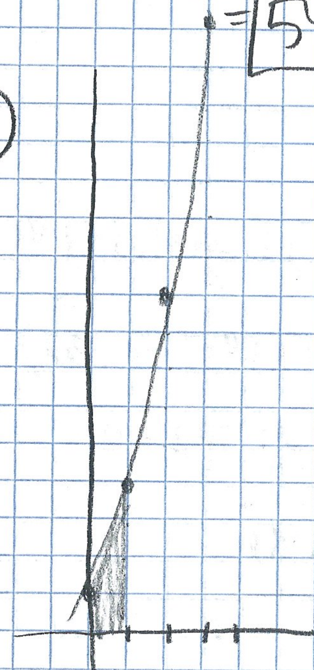
left $= \boxed{30}$

right $= 25+16+9+4$
$= \boxed{54}$

c)



d)



midpoint $= 1(2.25) + 1(6.25)$
$+ 1(12.25) + 1(20.25)$
$\cancel{+1(20.25)} = \boxed{41}$

trapezoid $= \frac{1}{2}(1)(1+4) + \frac{1}{2}(1)(4+9)$
$+ \frac{1}{2}(1)(9+16) + \frac{1}{2}(1)(16+25)$
$= \boxed{42}$

$$\int_0^4 (x+1)^2 = \frac{(x+1)^3}{3}\Big|_0^4 = \frac{(4+1)^3}{3} - \frac{(0+1)^2}{3} = \frac{125}{3} - \frac{1}{3} = \boxed{\frac{124}{3}}$$

#1.) $y=(x-1)^2$ $[0,4]$

x	y
0	1
1	0
2	1
3	4
4	9

#2) $y=x^2-1$ $[0,4]$

x	y
0	-1
1	0
2	3
3	8
4	15

#3) $\int_0^4 \sqrt{x}$ $y=\sqrt{x}$ $[0,4]$

x	y
0	0
1	1
2	1.414
3	1.732
4	2

#4) $y=(x+1)^2$ $[0,4]$

x	y
0	1
1	4
2	9
3	16
4	25