Name Answer key (1)
Period: _____

Precal: Review 7.1-7.2

Simplify the expression.

1. $\tan x \cdot \csc x$

2. $\dfrac{\sec x - \cos x}{\tan x}$

3. $\dfrac{\cot x}{\csc x - \sin x}$

4. $\dfrac{\sec x - \cos x}{\sin x}$

5. $\dfrac{\sec x}{\csc x}$

6. $\sin x \cdot \sec x$

Prove.

7. $\sin x + \cos x \cdot \cot x = \csc x$

8. $\dfrac{\sin x \cdot \sec x}{\tan x} = 1$

9. $\sec\left(\dfrac{\pi}{2} - u\right) = \csc u$

10. $\cos\left(x-\frac{\pi}{2}\right)=\sin x$

11. $\cos(x-y)+\cos(x+y)=2\cos x\cdot\cos y$

12. $\dfrac{\tan x}{\sec x}=\sin x$

13. $\dfrac{\cos x\cdot\sec x}{\tan x}=\cot x$

14. $(\sin x+\cos x)^2=1+2\sin x\cos x$

15. $\dfrac{\cos x}{\sec x}+\dfrac{\sin x}{\csc x}=1$

16. $\sin^2 x+\cos^2 x+\tan^2 x=\sec^2 x$

Use the addition or subtraction formula to find the exact value.

17. $\cos 75°$

18. $\sin 10° \cos 20° + \cos 10° \sin 20°$

19. $\cos \frac{11\pi}{12}$

20. $\sin 195°$

21. $\tan 15°$

22. $\frac{\tan 73° - \tan 13°}{1 + \tan 73° \tan 13°}$

① $\tan x \cdot \csc x$

$$\frac{\cancel{\sin x}}{\cos x} \cdot \frac{1}{\cancel{\sin x}} = \frac{1}{\cos x} = \boxed{\sec x}$$

② $\dfrac{\sec x - \cos x}{\tan x}$

$$\frac{\frac{1}{\cos x} - \frac{\cos x \cdot \cos x}{1 \cdot \cos x}}{\frac{\sin x}{\cos x}} = \frac{\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}}{\frac{\sin x}{\cos x}}$$

$$\sin^2 x + \cancel{\cos^2 x} = 1$$
$$\quad - \cancel{\cos^2 x} \quad -\cos^2 x$$
$$\sin^2 x = 1 - \cos^2 x$$

$$= \frac{\frac{1 - \cos^2 x}{\cos x}}{\frac{\sin x}{\cos x}}$$ flip

$$= \frac{\sin^2 x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{\cancel{\sin x}} = \boxed{\sin x}$$

(3) $\dfrac{\cot x}{\csc x - \sin x} = \dfrac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x} - \frac{\sin x \cdot \sin x}{1 \cdot \sin x}}$

$= \dfrac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}}$

$= \dfrac{\frac{\cos x}{\sin x}}{\boxed{\frac{1-\sin^2 x}{\sin x}}}$ Flip

$\sin^2 x + \cos^2 x = 1$
$\cancel{-\sin^2 x} \qquad -\sin^2 x$
$\cos^2 x = 1 - \sin^2 x$

$= \dfrac{\cancel{\cos x}}{\cancel{\sin x}} \cdot \dfrac{\cancel{\sin x}}{\cos^{\cancel{2}} x} = \dfrac{1}{\cos x} = \boxed{\sec x}$

(4) $\dfrac{\sec x - \cos x}{\sin x} = \dfrac{\frac{1}{\cos x} - \frac{\cos x \cdot \cos x}{1 \cdot \cos x}}{\sin x}$

$= \dfrac{\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x}}{\frac{\sin x}{1}}$ Flip

$= \dfrac{1-\cos^2 x}{\cos x} \cdot \dfrac{1}{\sin x} = \dfrac{\sin^{\cancel{2}} x}{\cos x} \cdot \dfrac{1}{\cancel{\sin x}} = \dfrac{\sin x}{\cos x} = \boxed{\tan x}$

⑤ $\dfrac{\sec x}{\csc x} = \dfrac{\frac{1}{\cos x}}{\frac{1}{\sin x}}$ Flip $= \dfrac{1}{\cos x} \cdot \dfrac{\sin x}{1} = \dfrac{\sin x}{\cos x} = \boxed{\tan x}$

⑥ $\sin x \cdot \sec x = \sin x \cdot \dfrac{1}{\cos x} = \dfrac{\sin x}{\cos x} = \boxed{\tan x}$

⑦ $\sin x + \cos x \cdot \cot x = \csc x$

$= \sin x + \boxed{\dfrac{\cos x}{1} \cdot \dfrac{\cos x}{\sin x}}$

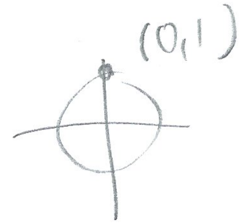
$= \dfrac{\sin x}{1} \cdot \dfrac{\sin x}{\sin x} + \dfrac{\cos^2 x}{\sin x} = \dfrac{\sin^2 x}{\sin x} + \dfrac{\cos^2 x}{\sin x} = \dfrac{\sin^2 x + \cos^2 x}{\sin x} = \dfrac{1}{\sin x}$

$= \boxed{\csc x}$

⑧ $\dfrac{\sin x \cdot \sec x}{\tan x} = 1$

$\dfrac{\frac{\sin x}{1} \cdot \frac{1}{\cos x}}{\frac{\sin x}{\cos x}} = \dfrac{\frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x}}$ Flip $= \dfrac{\sin x}{\cos x} \cdot \dfrac{\cos x}{\sin x} = \boxed{1}$

(9) $\sec\left(\frac{\pi}{2}-u\right) = \csc u$ $\qquad \cos(x-y) = \cos x \cdot \cos y + \sin x \cdot \sin y$



$$\frac{1}{\cos\left(\frac{\pi}{2}-u\right)} = \frac{1}{\cos\frac{\pi}{2}\cdot\cos u + \sin\frac{\pi}{2}\sin u}$$

$$= \frac{1}{0\cdot\cancel{\sin u} + 1\cdot\sin u} = \frac{1}{\sin u} = \boxed{\csc u}$$

(10) $\cos\left(x-\frac{\pi}{2}\right) = \sin x$

$\cos(s-t) = \cos s \cdot \cos t + \sin s \sin t$

$$\cos(\underset{s}{x}-\underset{t}{\tfrac{\pi}{2}}) = \cos x \cdot \cos\frac{\pi}{2} + \sin x \cdot \sin\frac{\pi}{2}$$

$$= \cancel{\cos x \cdot 0} + \sin x \cdot 1$$

$$= \boxed{\sin x}$$

(11) $\underbrace{\cos(x-y)}+\underbrace{\cos(x+y)} = 2\cos x\cdot\cos y$

$$\cos x\cdot\cos y+\cancel{\sin x\cdot\sin y} + \cos x\cdot\cos y-\cancel{\sin x\cdot\sin y}$$

$$\boxed{2\cos x\cdot\cos y}$$

(12) $\dfrac{\tan x}{\sec x} = \sin x$

$$\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \frac{\sin x}{\cancel{\cos x}}\cdot\frac{\cancel{\cos x}}{1} = \boxed{\sin x}$$

Flip

(13) $\dfrac{\cos x\cdot\sec x}{\tan x} = \cot x$

$$\frac{\cancel{\frac{\cos x}{1}}\cdot\frac{1}{\cancel{\cos x}}}{\frac{\sin x}{\cos x}} = \frac{1}{\frac{\sin x}{\cos x}} = 1\cdot\frac{\cos x}{\sin x} = \boxed{\cot x}$$

Flip

(14) $(\sin x+\cos x)^2 = 1+2\sin x\cdot\cos x$

$(\sin x+\cos x)(\sin x+\cos x)$

$\sin^2 x + \underbrace{\sin x\cdot\cos x + \sin x\cdot\cos x}+\cos^2 x$

$\underbrace{\sin^2 x+\cos^2 x} + 2\sin x\cdot\cos x$

$\boxed{1+2\sin x\cdot\cos x}$

(15) $\dfrac{\cos x}{\sec x}+\dfrac{\sin x}{\csc x}=1$

$\dfrac{\cos x}{\frac{1}{\cos x}}+\dfrac{\sin x}{\frac{1}{\sin x}}$

Flip Flip

$\cos x\cdot\dfrac{\cos x}{1}+\sin x\cdot\dfrac{\sin x}{1}$

$\cos^2 x+\sin^2 x=\boxed{1}$

(16) $\underbrace{\sin^2 x + \cos^2 x}_{} + \tan^2 x = \sec^2 x$

$1 + \tan^2 x = \boxed{\sec^2 x}$

by definition

(17) $\cos(75^\circ)$

$\cos(45^\circ + 30^\circ)$

$\cos(s+t) = \cos \overset{45}{s} \cdot \cos \overset{30}{t} - \sin \overset{45}{s} \cdot \sin \overset{30}{t}$

$= \cos 45^\circ \cdot \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

(18) $\sin \overset{s}{10} \cos \overset{t}{20} + \cos 10 \sin 20$

$\sin(s+t)$

$\sin(10+20)$

$\sin(30^\circ) = \boxed{\frac{1}{2}}$

(19) $\cos \frac{11\pi}{12}$ $\qquad \frac{11\pi}{\cancel{12}} \cdot \frac{\cancel{180}^{15}}{\pi} = 165^\circ$

$\cos(165^\circ) =$

$\cos(120 + 45)$

$\cos(\overset{120}{s} + \overset{45}{t}) = \cos \overset{120}{s} \cdot \cos \overset{45}{t} - \sin \overset{120}{s} \cdot \sin \overset{45}{t}$

$= \cos 120^\circ \cdot \cos 45^\circ - \sin 120^\circ \cdot \sin 45^\circ$

$= \frac{-1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{-\sqrt{2} - \sqrt{6}}{4}}$

(20) $\sin 195^\circ$

$\sin(150^\circ + 45^\circ)$

$\sin(s+t) = \sin \overset{150^\circ}{s} \cdot \cos \overset{45^\circ}{t} + \cos \overset{150^\circ}{s} \cdot \sin \overset{45}{t}$

$= \sin 150^\circ \cdot \cos 45^\circ + \cos 150^\circ \cdot \sin 45^\circ$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{-\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$

21 $\tan 15^\circ$

$\tan(45 - 30^\circ)$

$$\tan(s-t) = \frac{\tan \overset{45}{s} - \tan \overset{30}{t}}{1 + \tan \overset{45}{s} \cdot \tan \overset{30}{t}}$$

$$\tan 45^\circ = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \boxed{1}$$

$$\tan 30 = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \boxed{\frac{\sqrt{3}}{3}}$$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ}$$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 \cdot \frac{\sqrt{3}}{3}}$$

$$= \frac{\frac{1 \cdot 3}{1 \cdot 3} - \frac{\sqrt{3}}{3}}{\frac{1 \cdot 3}{1 \cdot 3} + \frac{\sqrt{3}}{3}} = \frac{\frac{3 - \sqrt{3}}{3}}{\frac{3 + \sqrt{3}}{3}}$$

$$= \frac{3 - \sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3 + \sqrt{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \frac{(3 - \sqrt{3})}{(3 - \sqrt{3})}$$

$$= \frac{9 - 3\sqrt{3} - 3\sqrt{3} + 3}{9 - \cancel{3\sqrt{3}} + \cancel{3\sqrt{3}} - 3}$$

$$= \frac{12^{\div 6} - 6^{\div 6}\sqrt{3}}{6_{\div 6}} = \boxed{2 - \sqrt{3}}$$

22 $\dfrac{\tan 73^\circ - \tan 13^\circ}{1+\tan 73^\circ \cdot \tan 13^\circ}$

$\tan(s-t) = \tan(73-13^\circ)$

$= \tan 60^\circ = \dfrac{y}{x} = \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$= \dfrac{\sqrt{3}}{\cancel{2}} \cdot \dfrac{\cancel{2}}{1} = \boxed{\sqrt{3}}$

Fundamental Trig Identities

Reciprocal Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

HALF-ANGLE FORMULAS

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

**The choice of the + or – sign depends on the quadrant in which $\frac{u}{2}$ lies

Addition and Subtraction Formulas

- Formulas for sin: $\sin(s+t) = \sin s \cdot \cos t + \cos s \cdot \sin t$
  $\sin(s-t) = \sin s \cdot \cos t - \cos s \cdot \sin t$
- Formulas for cosine: $\cos(s+t) = \cos s \cdot \cos t - \sin s \cdot \sin t$
  $\cos(s-t) = \cos s \cdot \cos t + \sin s \cdot \sin t$
- Formulas for tangent: $\tan(s+t) = \frac{\tan s + \tan t}{1 - \tan s \cdot \tan t}$
  $\tan(s-t) = \frac{\tan s - \tan t}{1 + \tan s \cdot \tan t}$

PRODUCT-TO-SUM FORMULAS

$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u+v) + \cos(u-v)]$$

$$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

Double Angle Formulas

$$\sin 2x = 2 \cdot \sin x \cdot \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

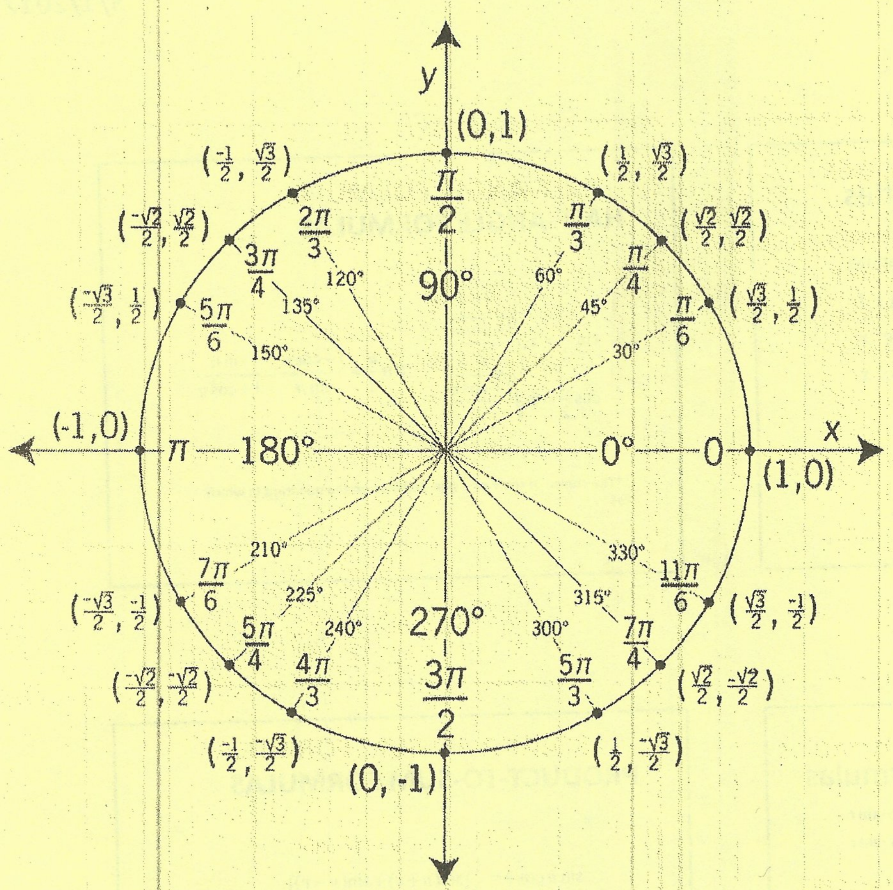
SUM-TO-PRODUCT FORMULAS

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$


y
x
(0,1)
(1,0)
(-1,0)
(0, -1)
π/2
90°
π
180°
0°
0
270°
3π/2
π/3
60°
π/4
45°
π/6
30°
2π/3
120°
3π/4
135°
5π/6
150°
7π/6
210°
5π/4
225°
4π/3
240°
5π/3
300°
7π/4
315°
11π/6
330°
(1/2, √3/2)
(√2/2, √2/2)
(√3/2, 1/2)
(-1/2, √3/2)
(-√2/2, √2/2)
(-√3/2, 1/2)
(-√3/2, -1/2)
(-√2/2, -√2/2)
(-1/2, -√3/2)
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