PreCalculus End of Course Review
Show all appropriate work for credit!!!

Name Answer key (!!)
Date/Period ____________

*1. Complete the following:

a. $\sin^2\theta + \underline{\cos^2\theta} = 1$ b. $\tan^2\theta + 1 = \underline{\sec^2\theta}$ c. $1 + \underline{\cot^2\theta} = \csc^2\theta$

*2. Graph each equation on the interval $[-2\pi, 2\pi]$

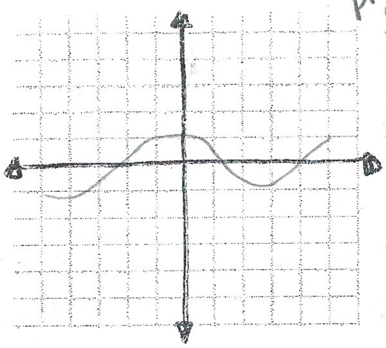
tan cot

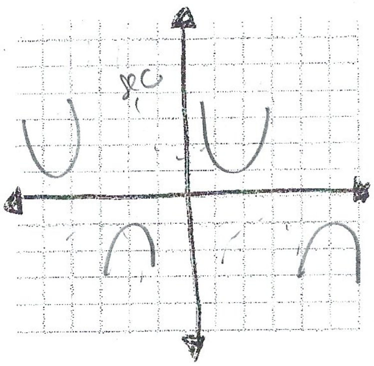
amp $= |-2| = 2$ period $= \frac{2\pi}{k} = \frac{2\pi}{2} = \boxed{\pi}$

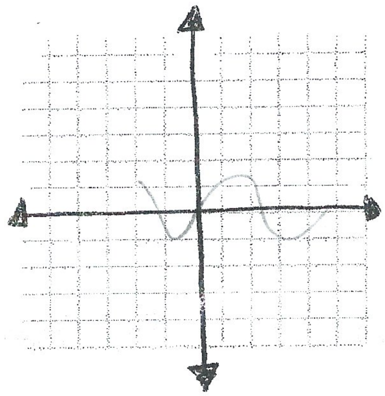
a. $f(x) = -2\cos 2\left(x - \frac{\pi}{4}\right) + 1$ up 1 b. $h(x) = \csc(x+\pi) - 1$ down 1 c. $y = \sin 3x$

flip

phase shift $\frac{\pi}{4}$ (right)







*3. Given the following functions, describe the amplitude, period, phase shift, and vertical shift.

a. $h(t) = -4\cos 3\left(t - \frac{\pi}{6}\right) - 1$ amp $= |-4| = 4$ period $= \boxed{\frac{2\pi}{3}}$ down 1 shift $\frac{\pi}{6}$ (right)

b. $g(t) = 2\tan\frac{1}{2}(t-\pi) - 3$ amp $= 2$ period $= \frac{\pi}{\frac{1}{2}} = \boxed{2\pi}$ shift π (right) down 3

c. $f(t) = 4\sec(2t + 4\pi) + 4$ $2(t+2\pi)$

d. $g(t) = 6\cos 3\left(t + \frac{\pi}{2}\right) + 3$

*4. Evaluate. Give exact answers (use radians).

a. $\sin\left(\frac{7\pi}{6}\right) = \boxed{-\frac{1}{2}}$ b. $\cos\left(\frac{7\pi}{6}\right) \boxed{-\frac{\sqrt{3}}{2}}$ c. $\tan\left(\frac{2\pi}{3}\right) = \frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \boxed{-\sqrt{3}}$ $(0,-1)$

d. $\sin 300° = \boxed{-\frac{\sqrt{3}}{2}}$ e. $\csc 150° = \frac{1}{y} = \frac{1}{\frac{1}{2}} = \boxed{2}$ f. $\cot(-45°) = \frac{x}{y} = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \boxed{-1}$ $315°$

g. $\cos\left(-\frac{2\pi}{3}\right)$ $\cos\left(\frac{4\pi}{3}\right) = \boxed{-\frac{1}{2}}$ h. $\tan\left(\frac{5\pi}{4}\right) = \frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \boxed{1}$ i. $\sec\left(\frac{\pi}{3}\right) = \frac{1}{x} = \frac{1}{\frac{1}{2}} = \boxed{2}$

*5. Evaluate. Give exact answers.

a. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $\sin x = -\frac{\sqrt{3}}{2}$ $\boxed{\frac{4\pi}{3}, \frac{5\pi}{3}}$

b. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ $\tan x = \frac{\sqrt{3}}{3}$ $y\ \frac{1}{2}\ \frac{2}{\sqrt{3}}$ $x\ \frac{\sqrt{3}}{2}$ $\boxed{\frac{\pi}{6}, \frac{7\pi}{6}}$

c. $\cos^{-1}\left(-\frac{1}{2}\right)$ $\cos x = -\frac{1}{2}$ $\boxed{\frac{2\pi}{3}, \frac{4\pi}{3}}$

d. $\tan^{-1}(-1)$ $\tan x = -1$ $\boxed{\frac{3\pi}{4}, \frac{7\pi}{4}}$

e. $\sin^{-1}\left(\cos\frac{2\pi}{3}\right)$ $\sin^{-1}\left(-\frac{1}{2}\right)$ $\sin x = -\frac{1}{2}$ $\boxed{\frac{7\pi}{6}, \frac{11\pi}{6}}$

f. $\cos^{-1}\left(\cos\frac{3\pi}{2}\right)$ $\cos^{-1}(0)$ $\cos x = 0$ $\boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$

Revised April 2010

195-180=15

*6. Evaluate. Give exact answer, use addition /subtraction identities:

a. $\cos 75^\circ$ $\cos(45^\circ+30^\circ) = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$

$\cos 45 \cos 30 - \sin 45 \sin 30$

b. neg. $\sin 195^\circ$ $(45-30)$

SOHCAH TOA

7.* Given $\sin\theta = -\frac{3}{5}$ and $\tan\theta > 0$, find

-4, 5, -3, θ

a. $\tan\theta = \boxed{\frac{3}{4}}$

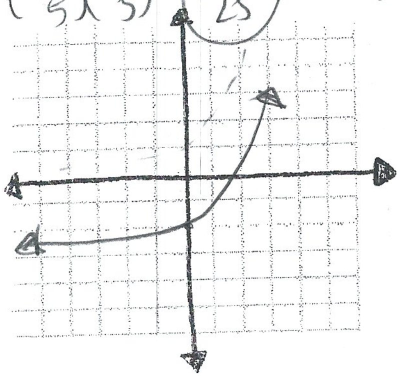
b. $\cos\theta = \boxed{-\frac{4}{5}}$

c. $\sin 2\theta$ $= 2\sin x\cos x$ $= 2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right) = \frac{24}{25}$

d. $\cos 2\theta$ $= \cos^2 x - \sin^2 x$ $= \frac{16}{25} - \frac{9}{25}$ $\boxed{\frac{7}{25}}$

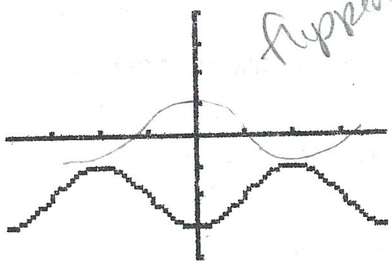
*8. Graph $f(x) = 2^x - 3$.

go through 1
then shift
down 3 units



*
9. Match the graph with correct function

a.

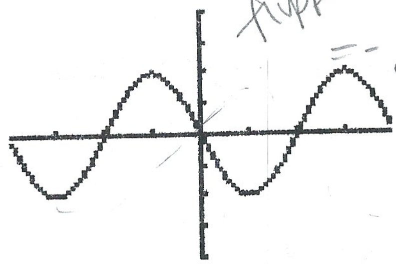


flipped

± shift up/down

a. ~~$f(x) = -3\cos x$~~
b. $g(x) = \sin x - 2$
c. ~~$h(x) = -3\tan x$~~
d. $f(x) = -\cos x - 2$

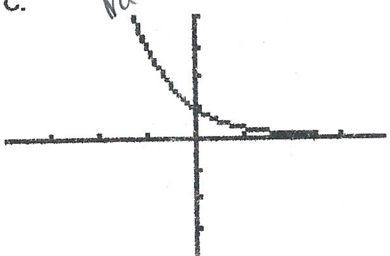
b.



flipped
$= -2\sin x$

a. $f(x) = 2\cos x$
b. $g(x) = -2\sin x$
c. ~~$h(x) = \tan x - 2$~~
d. $f(x) = -2\cos x$

c. Fraction



a. ~~$f(x) = 2^x$~~
b. $g(x) = \frac{1}{2}^x$
c. $h(x) = -2^x$
d. ~~$g(x0) = -\frac{1}{2}^x$~~ not flipped

d. regular #

normally goes through 1

a. $f(x) = 3^{x-1}$ more down
b. $g(x) = 3^x - 1$
c. ~~$h(x) = -3^x$~~
d. $f(x) = 3^{x+1}$ more up

• neg. in front flips to bottom

10. Evaluate.

a. $\log 10^3 + e^{\ln 5} - 5\ln e^2$

$3 + 5 - 5(2)$

$3+5-10 = \boxed{-2}$

b. $\ln e^5 - \log 1000 + 3^{\log_3 12}$

$5 - 3 + 12$

$2 + 12 = \boxed{14}$

c. $7^{\log_7 4} - e^{2\ln 5} - \log_3 27 = x$

$-e^{\ln 5^2}$

$4 - 5^2 - 3$

$4 - 25 - 3$

$4 - 28 = \boxed{-24}$

11. Expand:

a. $\log(15x^3yz^5)$

$\boxed{\log 15 + 3\log x + \log y + 5\log z}$

b. $\ln\left(\dfrac{25a^3b}{c^4}\right)$

$\boxed{2\ln 5 + 3\ln a + \ln b - 4\ln c}$

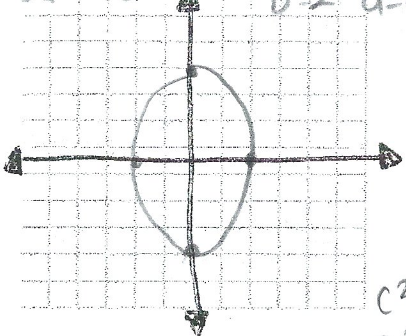
12. Graph the ellipse and state the center, vertices and foci.

$9x^2 + 4y^2 = 36$

$\frac{9x^2}{36} + \frac{4y^2}{36} = \frac{36}{36}$

$\frac{x^2}{4} + \frac{y^2}{9} = 1$

$b=2 \quad a=3$



center $(0,0)$

vertices $(0, \pm 3)$

foci $(0, \pm\sqrt{5})$

$c^2 = a^2 - b^2$

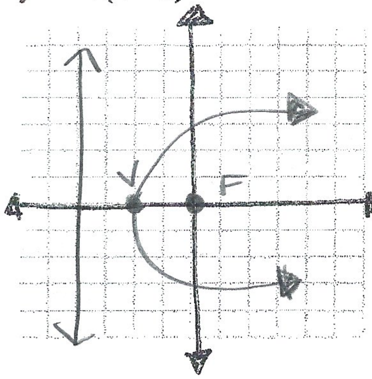
$c^2 = 9 - 4 = \sqrt{5}$

13. Graph the parabola and state the vertex, focus, and directrix.

$4p = 8$

$p = 2$ x-axis

$y^2 = 8(x+2)$



Vertex $(-2, 0)$

Focus $(-2+2, 0)$

$(0, 0)$

Directrix

$x = -2-2$

$x = -4$

14. Graph the hyperbola and state the center, vertices, foci, and equations of the asymptotes.

$\frac{y^2}{\sqrt{4}} - \frac{x^2}{\sqrt{20}} = 1$

$a=2 \quad b = 2\sqrt{5}$

$c^2 = a^2 + b^2$

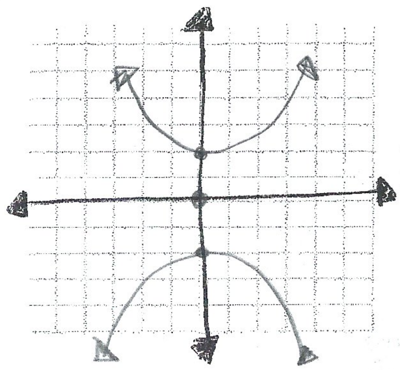
$c^2 = 4 + 20 = \sqrt{24} = 2\sqrt{6}$

center $(0,0)$

vertices $(0, \pm 2)$

foci $(0, \pm 2\sqrt{6})$

asymptotes $y = \pm \frac{2}{2\sqrt{5}}x$



15. Find the equation of the conic section, a parabola, with a vertex at $(2, 1)$ and focus at $(2, -1)$.

change y

$\boxed{(x-2)^2 = -8(y-1)}$

$y = -\frac{1}{8}(x-2)^2 + 1 \quad -2 \quad p = -2$

16. Identify the shapes of the graphs of the following equations:

a. $2x^2 + 3y + 7x + 9 = 0$

parabola

b. $5x^2 + 5y^2 + 3x + 9y + 12 = 0$

ellipse

c. $5x^2 + 6y^2 + 7x + 8y + 11 = 0$

ellipse

d. $10x^2 - 7y^2 + 6x + 9y + 4 = 0$

hyperbola

17. Solve the following system of equations:

$$x + 2y + z = 1$$
$$y + 2z = 5$$
$$x + y + 3z = 8$$

plug in answers

18. Find the product, if it exists:

a. $\underset{2\times 2}{\begin{bmatrix} 1 & 3 \\ -4 & 0 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} 2 & 5 & -2 \\ 3 & 0 & 4 \end{bmatrix}} = \begin{bmatrix} 11 & 5 & 10 \\ -8 & -20 & 8 \end{bmatrix}$

b. $\underset{3\times 2}{\begin{bmatrix} 2 & 1 \\ -2 & 5 \\ 3 & -3 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 2 \\ 5 \end{bmatrix}} = \begin{bmatrix} 9 \\ 21 \\ -9 \end{bmatrix}$

c. $\underset{2\times 2}{\begin{bmatrix} -1 & 2 \\ 4 & 9 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} -5 & 5 \\ 0 & 1 \\ 2 & 7 \end{bmatrix}} = \boxed{\text{does not exist}}$

d. $\begin{bmatrix} 1 & 2 & 0 \\ -1 & 4 & -3 \\ 5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 2 \\ 3 & -2 & 4 \\ 5 & 1 & 1 \end{bmatrix} =$

19. Find the determinant of the matrix.

a. $\begin{bmatrix} 5 & 7 \\ 1 & 4 \end{bmatrix}$ $20 - 7 = \boxed{13}$

b. $\begin{bmatrix} -3 & 3 \\ 8 & 5 \end{bmatrix}$ $-15 - 24 = \boxed{-39}$

c. $\begin{bmatrix} 1 & 3 \\ -3 & -1 \end{bmatrix}$ $-1 + 9 = \boxed{8}$

20. Find A^{-1} of each matrix if it exists

a. $A = \begin{bmatrix} 3 & 8 \\ 2 & 6 \end{bmatrix}$ $\frac{1}{18-16} = \frac{1}{2}\begin{bmatrix} 6 & -8 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ -1 & \frac{3}{2} \end{bmatrix}$

b. $A = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$ $\frac{1}{3+8} = \frac{1}{11}\begin{bmatrix} 1 & 2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{11} & \frac{2}{11} \\ -\frac{4}{11} & \frac{3}{11} \end{bmatrix}$

21. Find the sum: $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_n = n\left(\frac{a_1 + a_n}{2}\right)$

a. $\sum_{k=1}^{6} (6 - 3k) = 6\left(\frac{3 + -12}{2}\right) = \cancel{6}^{3}\left(\frac{-9}{\cancel{2}}\right) = \boxed{-27}$

$a_1 = 6 - 3 = 3$
$a_6 = 6 - 18 = -12$

b. $\sum_{n=1}^{4} 2(3)^{n-1} = \frac{4}{2}[4 + (4-1)(3)] = 2[4 + 3(3)] = 2[13) = \boxed{26}$

$a_1 = 2(3)^0 = 2$

c. $\sum_{k=1} 4\left(\frac{1}{2}\right)^k$ $\frac{5}{2}[4 + (5-1)(\frac{1}{2})]$ $= \frac{5}{2}[4 + 2] = \frac{5}{\cancel{2}}\cdot\cancel{6}^{3} = \boxed{15}$

$a_1 = 4(\frac{1}{2}) = 2$

d. $\sum_{k=0}^{5} 5\left(\frac{2}{3}\right)^k = \frac{5}{2}[10 + (5-1)(\frac{2}{3})]$ $= \frac{5}{2}[10 + 4 \cdot \frac{2}{3}] \ldots$

$a_1 = 5$

22. According to the Fundamental Theorem of Algebra, how many zeros do the following polynomials have in the complex number system?

a. $x^3 + 2x^2 + x + 5 = 0$ $\boxed{2}$

b. $7x^8 + 9 = 0$ $\boxed{2}$

c. $6x^4 + 3x^3 + 2x = 0$ $\boxed{2}$

23. Find the average rate of change of the function $f(x) = x^2 + 3x - 5$ from $x = -2$ to $x = 5$. $\frac{f(b) - f(a)}{b - a}$

$\frac{f(5) - f(-2)}{5 + 2} = \frac{5^2 + 3(5) - 5 - [(-2)^2 + 3(-2) - 5]}{7} = \frac{35 + 7}{7} = \frac{42}{7} = \boxed{6}$

$\frac{f(b)-f(a)}{b-a} = \frac{f(x+h)-f(x)}{x+h-x} = \frac{5(x+h)-2-[5x-2]}{h}$

24\. Find the difference quotient of the function $f(x) = 5x - 2$ $= \frac{\cancel{5x}+5h\cancel{-2}\cancel{-5x}\cancel{+2}}{h} = \frac{5h}{h} = \boxed{5}$

*

25\. Find a complete graph of the following function, identifying all vertical and horizontal asymptotes and holes. holes = common

hole $x=-3$

$f(x) = \frac{(\cancel{x+3})(2x)}{(\cancel{x+3})(x-1)}$

vertical asy

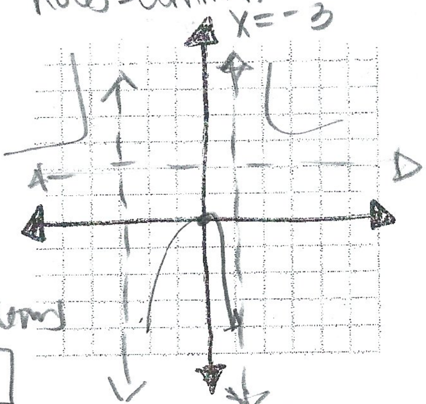
bottom = 0

$x-1=0$

$\boxed{x=1}$

horizontal leading terms

$y = \frac{2x}{x}$ $\boxed{y=2}$



26\. Find a complete graph of the following function, identifying all vertical and horizontal asymptotes and holes

hole

$x+1=0$ $\boxed{x=-1}$

$g(x) = \frac{x^2+x}{x^2-2x-3}$

hole $= -1$

$= x(\cancel{x+1})$

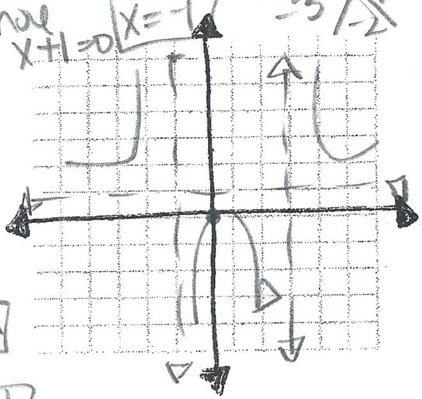
$(x-3)(\cancel{x+1})$

vertical

$\boxed{x=3,}$

horizontal

$y = \frac{x^2}{x^2}$ $\boxed{y=1}$



*

27\. Factor and simplify: $\tan^2 x - \tan^2 x \sin^2 x$

$\tan^2 x(1-\sin^2 x) = \tan^2 x(\cos^2 x) = \frac{\sin^2 x}{\cancel{\cos^2 x}} \cdot \cancel{\cos^2 x} = \boxed{\sin^2 x}$

28\. Simplify:

a. $\sin 35° \cos 50° - \cos 35° \sin 50°$ calc

$\sin(35-50) = \sin(-15°) = \sin(30-45°)$ plug in

b. $\cos\left(\frac{\pi}{5}\right)\cos\left(\frac{\pi}{9}\right) + \sin\left(\frac{\pi}{5}\right)\sin\left(\frac{\pi}{9}\right)$

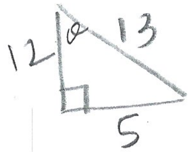
$\cos\left(\frac{\pi}{5}-\frac{\pi}{9}\right) = \cos\left(\frac{4\pi}{45}\right)$ calc.

29\. Given $\cos\theta = \frac{4}{5}$, find $\cos\left(\frac{\theta}{2}\right)$. (Assume $0 < \theta < \frac{\pi}{2}$)

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos x}{2}} = \pm\sqrt{\frac{1-\frac{4}{5}}{2}} = +\sqrt{\frac{\frac{1}{5}}{\frac{2}{1}}} \cdot \frac{1}{2} = +\frac{1}{\sqrt{10}} = \boxed{\frac{\sqrt{10}}{10}}$

30\. Given $\sin\theta = -\frac{5}{13}$, find $\sin(2\theta)$. (Assume $\pi < \theta < \frac{3\pi}{2}$)

$\sin 2\theta = 2\sin x\cos x = 2\left(\frac{-5}{13}\right)\left(\frac{-12}{13}\right) = \boxed{\frac{120}{169}}$



31\. Multiply and simplify the following:

a. $(1+\cos x)(1-\cos x)$

$1 - \cancel{\cos x} + \cancel{\cos x} - \cos^2 x$

$1 - \cos^2 x$

$\boxed{\sin^2 x}$

b. $(1+\sin x)(1-\sin x)$

$1 - \cancel{\sin x} + \cancel{\sin x} - \sin^2 x$

$1 - \sin^2 x$

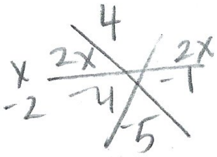
$\boxed{\cos^2 x}$

c. $(\cos x + \sin x)^2$

$(\cos x + \sin x)(\cos x + \sin x)$

$\cos^2 x + 2\sin x\cos x + \sin^2 x$

$\boxed{1 + 2\sin x\cos x}$



*32. Solve the following over $[0, 2\pi)$

a. $\cos x \sin x - \sin x = 0$

$\sin x(\cos x - 1) = 0$

$\sin x = 0 \quad \cos = 1 \quad (0, 2\pi)$

b. $2\sin^2 x - 5\sin x + 2 = 0$

$(\sin x - 2)(2\sin x - 1) = 0$

$\cancel{\sin x = 2} \quad \sin x = \frac{1}{2} \quad \boxed{\frac{\pi}{6}, \frac{5\pi}{6}}$

c. $\sin x = \cos(2x)$

$\cancel{\sin x = \cos^2 - \sin^2 x}$

*33. Convert to radians.

a. $36° \cdot \frac{\pi}{180} = \boxed{\frac{\pi}{5}}$

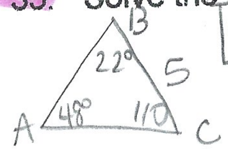
b. $275° \cdot \frac{\pi}{180} = \frac{275\pi}{180} \div 5 = \boxed{\frac{55\pi}{36}}$

*34. Convert to degrees.

a. $\frac{5\pi}{12} \cdot \frac{180}{\pi} = \boxed{75°}$

b. $\frac{11\pi}{9} \cdot \frac{180}{\pi} = \boxed{220°}$

*35. Solve the triangle given $A = 48°$ $B = 22°$ $a = 5$.

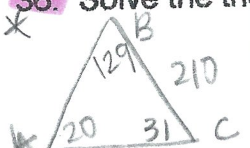


$\boxed{\angle C = 110}$

$\frac{\sin A}{a} = \frac{\sin B}{b}$ $\quad \frac{\sin 48}{5} = \frac{\sin 22}{b}$ $\quad \boxed{b = 2.5}$

$\frac{\sin A}{a} = \frac{\sin C}{c}$ $\quad \frac{\sin 48}{5} = \frac{\sin 110}{c}$ $\quad \boxed{C = 6.3}$

36. Solve the triangle with $A = 20°$, $C = 31°$, and $a = 210$ *



$\boxed{\angle B = 129}$

$\frac{\sin A}{a} = \frac{\sin B}{b}$ $\quad \frac{\sin 20}{110} = \frac{\sin 129}{b}$ $\quad \boxed{b = 250}$

$\frac{\sin A}{a} = \frac{\sin C}{c}$ $\quad \frac{\sin 20}{110} = \frac{\sin 31}{c}$ $\quad \boxed{C = 165.6}$

*37. Convert the polar coordinates $\left(4, \frac{7\pi}{6}\right)$ to rectangular coordinates.

$x = r\cos\theta$

$x = 4\cos\left(\frac{7\pi}{6}\right) = 4\left(-\frac{\sqrt{3}}{2}\right) = \boxed{-2\sqrt{3}}$

$y = r\sin\theta$

$= 4\sin\left(\frac{7\pi}{6}\right) = 4\left(-\frac{1}{2}\right) = \boxed{-2}$

*38. Convert the rectangular coordinates $(2\sqrt{3}, -2)$ to polar coordinates.

$r^2 = x^2 + y^2$

$r^2 = (2\sqrt{3})^2 + (-2)^2 = 12 + 4 = \sqrt{16} \quad \boxed{r = 4}$

$\tan \frac{-2}{2\sqrt{3}} \quad \tan -\frac{1}{\sqrt{3}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \boxed{\frac{11\pi}{6}}$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

*39. Given $u = 7i - j$, $v = 2i + j$ and $w = 6i + 2j$

$u = \langle 7, -1 \rangle \quad v = \langle 2, 1 \rangle \quad w = \langle 6, 2 \rangle$

a. $5u - 2v$

$5\langle 7, -1 \rangle - 2\langle 2, 1 \rangle = \langle 35, -5 \rangle + \langle -4, -2 \rangle$ $\quad \boxed{\langle 31, -7 \rangle}$

b. $u \bullet v$ $\quad 14 + -1 = \boxed{13}$

c. $u \bullet (v + w)$

$\langle 7, -1 \rangle \bullet \langle 8, 3 \rangle = 56 + -3 = \boxed{53}$

d. $(u - v) \bullet (v + w)$

$\langle 5, -2 \rangle \cdot \langle 8, 3 \rangle = 40 + -6 = \boxed{34}$

*40. Write as a single logarithm.

a. $2\log x - \log(x+3) + \log(x^2 - 9)$

$\log x^2 - \log(x+3) + \log(x^2 - 9)$

$\log \frac{x^2}{x+3} \cdot (x^2 - 9)$

$\log \frac{x^2}{x+3} \cdot (x+3)(x-3) = \boxed{\log x^2(x-3)}$

b. $3\ln 5x - 4\ln(x-5) + 2\ln(x-5)$

$\ln(5x)^3 - \ln(x-5)^4 + \ln(x-5)^2$

$\ln \frac{125x^3}{(x-5)^4} \cdot (x-5)^2 = \boxed{\ln \frac{125x^3}{(x-5)^2}}$

41. Use the following matrices to find:

$A=\begin{bmatrix}2 & 3 & -1\\ 5 & 0 & 4\end{bmatrix}$ $B=\begin{bmatrix}2 & -3 & 4\\ 1 & 5 & 7\end{bmatrix}$ $C=\begin{bmatrix}-2 & 6\\ 1 & 4\end{bmatrix}$

$D=\begin{bmatrix}2 & 0\\ -1 & 3\end{bmatrix}$

a. $2A+B$

$\begin{bmatrix}4 & 6 & -2\\ 10 & 0 & 8\end{bmatrix}+\begin{bmatrix}2 & -3 & 4\\ 1 & 5 & 7\end{bmatrix}=\boxed{\begin{bmatrix}6 & 3 & 2\\ 11 & 5 & 15\end{bmatrix}}$

b. $-3A+2B$

$\begin{bmatrix}-6 & -9 & 3\\ -15 & 0 & -12\end{bmatrix}+\begin{bmatrix}4 & -6 & 8\\ 2 & 10 & 14\end{bmatrix}=\boxed{\begin{bmatrix}-2 & -15 & 11\\ -13 & 10 & 2\end{bmatrix}}$

c. $\frac{1}{2}C+2D$

$\begin{bmatrix}-1 & 3\\ \frac{1}{2} & 2\end{bmatrix}+\begin{bmatrix}4 & 0\\ -2 & 6\end{bmatrix}$

$\boxed{\begin{bmatrix}3 & 3\\ -\frac{3}{2} & 8\end{bmatrix}}$

42. Give the dimensions of each matrix.

a. $\begin{bmatrix}3 & 4 & -1 & 7\end{bmatrix}$ $\boxed{1\times4}$

b. $\begin{bmatrix}2 & 5 & 1\\ 0 & 6 & 3\end{bmatrix}$ $\boxed{2\times3}$

c. $\begin{bmatrix}5 & -2\\ 8 & 6\\ 4 & 0\end{bmatrix}$ $\boxed{3\times2}$

43. Solve.

a. $(3x-2y=4)\cdot-2$ = $-6x+4y=-8$
$6x-4y=8$ $\quad 6x-4y=8$

$\boxed{\text{infinitely many}}$

b. $2x-5y=4$
$x+y-z=8$
$3x+5z=0$

plug in answers

44. Is the system of equations, $\begin{array}{l}3x+5y=19\\ -3x+y=3\end{array}$ consistent, inconsistent?

$6y=16$

How many solutions are there to the system (one, none or infinitely many)?

one

45. Simplify and write your results in the form $a+bi$.

a. $7+\sqrt{-12}$

$\boxed{7+2i\sqrt{3}}$

b. $(3+4i)^2$

$9+24i+16i^2$

$9+24i-16=\boxed{-7+24i}$

c. $2(3+5i)-(2-2i)$

$6+10i-2+2i$

$\boxed{4+12i}$

d. $3\sqrt{-4}+6\sqrt{-36}$

$3\cdot2i+6\cdot6i=6i+36i=\boxed{42i}$

e. $(2-4i)(3+5i)$

$6+10i-12i-20i^2$ $+20$ $\boxed{26-2i}$

f. $2(1+4i)+4(2-5i)$

$2+8i+8-20i$

$\boxed{10-12i}$

46. Find the sum of the infinite geometric series, if it exists.

$S=\frac{a}{1-r}$

a. $\sum_{n=1}^{\infty}\left(\frac{3}{5}\right)^n=\frac{\frac{3}{5}}{1-\frac{3}{5}}=\frac{\frac{3}{5}}{\frac{2}{5}}$ $\boxed{\frac{3}{2}}$

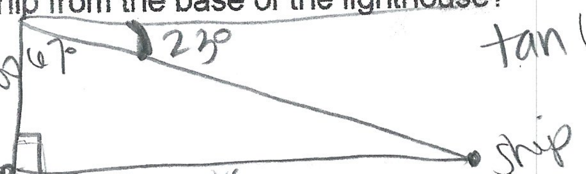
b. $\sum_{n=1}^{\infty}3(4)^n=\frac{12}{1-4}=\frac{12}{-3}$ $\boxed{4}$

c. $\sum_{n=1}^{\infty}2\left(\frac{1}{3}\right)^{-(n+1)}$ $2\left(\frac{1}{3}\right)^{-2}$ $=\frac{18}{1-\frac{1}{3}}=\frac{18}{\frac{2}{3}}$ $\boxed{27}$

d. $\sum_{n=1}^{\infty}5(3)^{-(2n)}$ $5(3^{-2})=\frac{5}{9}$ $=\frac{\frac{5}{9}\cdot\frac{1}{3}}{3}$ $\boxed{\frac{5}{27}}$

47. From the top of a 200 foot lighthouse, the angle of depression to a ship in the ocean is 23°. How far is the ship from the base of the lighthouse?

$90-23=67$



$\tan 67°=\frac{x}{200}$

$x=200\cdot\tan 67°$

$x=471.2$

PRECALCULUS FORMULAS

Nth Term Formulas

$a_n = a_1 + (n-1)d$, arithmetic

$a_n = a_1 r^{n-1}$, geometric

Summation Formulas

Arithmetic: $S_n = \sum_{k=1}^{n} a_k = n\left(\frac{a_1 + a_n}{2}\right)$

Finite Geometric: $S_n = \sum_{k=1}^{n} a_k = a\left(\frac{1-r^n}{1-r}\right)$

Infinite Geometric: $S_n = \sum_{k=1}^{\infty} a_k = \frac{a_1}{1-r}$,

Addition/Subtraction Identities

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

Double Angle Identities

$\sin 2x = 2\sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

Half Angle Identities

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$

Average Rate of Change

$$\frac{f(b)-f(a)}{b-a}$$

Difference Quotient

$$\frac{f(x+h)-f(x)}{h}$$

Polar to Rectangular and Rectangular to Polar

$x = r\cos\theta \quad y = r\sin\theta$

$r^2 = x^2 + y^2 \quad \tan\theta = \frac{y}{x}$

Standard Form Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Standard Form-Hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc\cos A$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of a Triangle

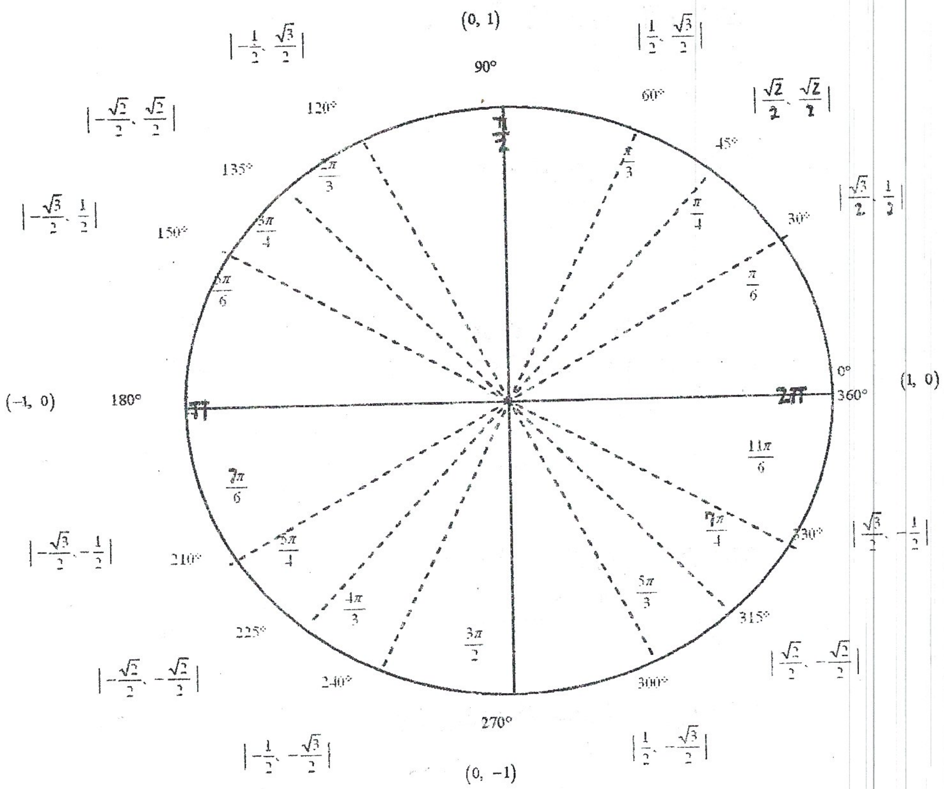
$Area = \frac{1}{2}bc\sin A$

Heron's Formula

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{1}{2}(a+b+c)$

Unit Circle



(0, 1)
90°
$\frac{\pi}{2}$
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
60°
$\frac{\pi}{3}$
$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
45°
$\frac{\pi}{4}$
$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
30°
$\frac{\pi}{6}$
0°
2π
360°
(1, 0)
$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
120°
$\frac{2\pi}{3}$
$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
135°
$\frac{3\pi}{4}$
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
150°
$\frac{5\pi}{6}$
(−1, 0)
180°
π
$\frac{7\pi}{6}$
$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
210°
$\frac{5\pi}{4}$
225°
$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\frac{4\pi}{3}$
240°
$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$\frac{3\pi}{2}$
270°
(0, −1)
$\frac{5\pi}{3}$
300°
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
$\frac{7\pi}{4}$
315°
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\frac{11\pi}{6}$
330°
$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$