Name: Answer key Class: ____________ Date: ________ ID: A

Pre-Calculus: Chapter#9 Review

1. Use the substitution method to find all solutions of the system of equations.

$$\begin{cases} 2x + y = 11 \\ -2(x + 2y = 1) \end{cases}$$

$2x + y = 11$
$-2x - 4y = -2$

$-3y = 9$
$\boxed{y = -3}$

$2x + -3 = 11$
$+3 \quad +3$
$2x = 14$
$\boxed{x = 7}$

$(7, -3)$

2. Use the substitution method to find all solutions of the system of equations.

$$\begin{cases} 2x^2 + y^2 = 3 \\ x + y = 0 \end{cases}$$

solve for x or y
$x + y = 0$
$-y \quad -y$
$x = -y$

plug in
$2x^2 + y^2 = 3$
$2(-y)^2 + y^2 = 3$
$2y^2 + y^2 = 3$
$3y^2 = 3$
$\sqrt{y^2} = \sqrt{1}$
$\boxed{y = \pm 1}$

If $y = 1$
$x + y = 0$
$x + 1 = 0$
$x = -1$
$(-1, 1)$

If $y = -1$
$x + y = 0$
$x + -1 = 0$
$\boxed{x = 1}$
$(1, -1)$

3. determine if the system has one solution, no solution, or infinitely many solutions.

$$\begin{cases} 7(6x + 18y = 0) \\ 6(-7x - 21y = 84) \end{cases}$$

$42x + 126y = 0$
$-42x - 126y = 504$

$0 = 504$
$\boxed{\text{No solution}}$

4. Find the complete solution of the linear system, or show that it is inconsistent.

$$\begin{cases} x + y - 2z = 2 \\ 3x - y - 5z = 8 \\ 2x + y + 2z = -7 \end{cases}$$

cancel out y

$3x - y - 5z = 8$
$2x + y + 2z = -7$

$5x - 3z = 1$

$x + y - 2z = 2$
$3x - y - 5z = 8$

$4x - 7z = 10$

$4(5x - 3z = 1)$
$-5(4x - 7z = 10)$

$20x - 12z = 4$
$-20x + 35z = -50$

$\frac{23z}{23} = \frac{-46}{23}$
$\boxed{z = -2}$

$5x - 3z = 1$
$5x - 3(-2) = 1$
$5x + 6 = 1$
$-6 \quad -6$
$\frac{5x}{5} = \frac{-5}{5}$
$\boxed{x = -1}$

$(-1, -1, -2)$

$x + y - 2z = 2$
$-1 + y - 2(-2) = 2$
$-1 + y + 4 = 2$
$y + 3 = 2$
$-3 \quad -3$
$\boxed{y = -1}$

$\{(4,3,4)\}$

5. The system of linear equations has a unique solution. Find the solution.

cancel out y

$$\begin{cases} x + y + z = 11 \\ 2x - 3y + 2z = 7 \\ 4x + y - 3z = 7 \end{cases}$$

$3(x+y+z=11)$
$2x-3y+2z=7$

$3x+3y+3z=33$
$2x-3y+2z=7$

$5x+5z=40$

$2x-3y+2z=7$
$3(4x+y-3z=7)$

$2x-3y+2z=7$
$12x+3y-9z=21$

$14x-7z=28$

$5(14x-7z=28)$
$7(5x+5z=40)$

$70x-35z=140$
$35x+35z=280$

$\frac{105x}{105}=\frac{420}{105}$
$\boxed{x=4}$

$5x+5z=40$
$5(4)+5z=40$
$20+5z=40$
$-20 \quad -20$
$\frac{5z}{5}=\frac{20}{5}$
$\boxed{z=4}$

$x+y+z=11$
$4+y+4=11$
$y+8=11$
$-8 \quad -8$
$\boxed{y=3}$

6. Solve for x and y.

$$2\begin{bmatrix} x & y \\ x+y & x-y \end{bmatrix} = \begin{bmatrix} 4 & -8 \\ -4 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 2x & \boxed{2y} \\ 2x+2y & 2x-2y \end{bmatrix} = \begin{bmatrix} 4 & \boxed{-8} \\ -4 & 12 \end{bmatrix}$$

$\frac{2x}{2}=\frac{4}{2}$ $\boxed{x=2}$ $\frac{2y}{2}=\frac{-8}{2}$ $\boxed{y=-4}$

$x =$ 2, $y =$ -4

7. Perform the matrix operation, or if it is impossible, explain why.

$$\begin{bmatrix} 0 & 1 & 3 \\ 2 & 1 & 0 \end{bmatrix} - \begin{bmatrix} 9 & 1 & -3 \\ 2 & 9 & -9 \end{bmatrix} = \begin{bmatrix} 0-9 & 1-1 & 3++3 \\ 2-2 & 1-9 & 0++9 \end{bmatrix} = \begin{bmatrix} -9 & 0 & 6 \\ 0 & -8 & 9 \end{bmatrix}$$

8. Perform the matrix operation, or if it is impossible, explain why.

2X2 2X3

$$\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & -2 & 3 \\ 2 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1+4 & -2+4 & 3+-2 \\ -1+8 & 2+8 & -3+-4 \end{bmatrix} = \begin{bmatrix} 5 & 2 & 1 \\ 7 & 10 & -7 \end{bmatrix}$$

9. Solve the matrix equation for the unknown matrix X, or explain why no solution exists.

$A = \begin{bmatrix} 6 & 4 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 3 \\ 5 & 6 \end{bmatrix}$

$2X + A = B$

$-A \quad -A$

$\frac{\cancel{2}X}{\cancel{2}} = \frac{B-A}{2}$

$X = \frac{1}{2}(B-A)$

$\frac{1}{2}\left(\overset{B}{\begin{bmatrix} 4 & 3 \\ 5 & 6 \end{bmatrix}} - \begin{bmatrix} 6 & 4 \\ 1 & 2 \end{bmatrix}\right)$

$\frac{1}{2}\begin{bmatrix} -2 & -1 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} -1 & -\frac{1}{2} \\ 2 & 2 \end{bmatrix}$

10. Find the inverse of the matrix.

$\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ 15 16

switch signs

switch place

$= \frac{1}{16-15}\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$

$= \frac{1}{1}\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$

11. Solve the system of equations ~~by converting to a matrix equation and using the inverse of the coefficient matrix.~~

$4(5x + 4y = -2)$

$-5(4x + 3y = 1)$

$\cancel{20x} + 16y = -8$

$-\cancel{20x} - 15y = -5$

$\boxed{y = -13}$

$5x + 4y = -2$

$5x + 4(-13) = -2$

$5x - \cancel{52} = -2$

$+\cancel{52} \quad +52$

$\frac{\cancel{5}x}{\cancel{5}} = \frac{50}{5}$

$\boxed{x = 10}$

$(10, -13)$

12. Find the determinant of the matrix if it exists.

$$\begin{bmatrix} -3 & 1 \\ 5 & -4 \end{bmatrix} \quad \begin{matrix} 5 \\ 12 \end{matrix}$$

$12 - 5 = \boxed{7}$

13. Find the determinant of the matrix.

$$A = \begin{bmatrix} 10 & 0 & 50 \\ 60 & 50 & 60 \\ 40 & 10 & 50 \end{bmatrix}$$

14. ~~Use Cramer's Rule to~~ solve the system.

$$-2\left(4x + 5y = 80\right)$$
$$8x + 8y = 144$$

$$-8x - 10y = -160$$
$$\cancel{8x} + 8y = 144$$

$$\frac{-2y}{-2} = \frac{-16}{-2}$$

$$\boxed{y = 8}$$

$$4x + 5y = 80$$
$$4x + 5(8) = 80$$
$$4x + 40 = 80$$
$$-40 \quad -40$$
$$\frac{4x}{4} = \frac{40}{4}$$

$$\boxed{x = 10}$$

$(10, 8)$