Pre-calculus



1

## Section#2.1
## What is a Fundtion?

Standard: Algebra 2 Review

2

**Example:** Complete the table.

$g(x) = |2x+3|$

| x | g(x) |
|---|---|
| -3 | 3 |
| -2 | 1 |
| 0 | 3 |
| 1 | 5 |
| 3 | 9 |

$g(-3) = |2(-3)+3| = |-6+3| = |-3| = \boxed{3}$

$g(-2) = |2(-2)+3| = |-4+3| = |-1| = \boxed{1}$

$g(0) = |2(0)+3| = |3| = \boxed{3}$

$g(1) = |2(1)+3| = |5| = \boxed{5}$

$g(3) = |2(3)+3| = |9| = \boxed{9}$

3

**Example:** Evaluate the function at the indicated values.

$f(x) = x^2 + 2x$ ; f(0), f(3), f(-3), f(a), f(-x)

$f(0) = 0^2 + 2(0) = \boxed{0}$

$f(3) = 3^2 + 2(3) = 9+6 = \boxed{15}$

$f(-3) = (-3)^2 + 2(-3) = 9-6 = \boxed{3}$

$f(a) = \boxed{a^2 + 2a}$

$f(-x) = (-x)^2 + 2(-x) = \boxed{x^2 - 2x}$

4

**Example:** Evaluate the piecewise defined at the indicated values.

$$f(x) = \begin{cases} 5 & \text{if } x \le 2 \\ 2x-3 & \text{if } x > 2 \end{cases}$$

f(-3), f(0), f(2), f(3), f(5)

$f(-3) = \boxed{5}$

$f(0) = \boxed{5}$

$f(2) = \boxed{5}$

$f(3) = 2(3) - 3 = 6 - 3 = \boxed{3}$

$f(5) = 2(5) - 3 = 10 - 3 = \boxed{7}$

5

**Example:** Evaluate and simplify.

f(x)=3x-1; f(2x) and 2f(x)

$f(2x) = 3(2x) - 1 = \boxed{6x-1}$

$2f(x) = 2(3x-1) = \boxed{6x-2}$

6

**Example:** Find f(a), f(a+h) and $\frac{f(a+h)-f(a)}{h}$

$f(x) = x^2 + 1$

$f(a) = \boxed{a^2+1}$

$f(a+h) = (a+h)^2 + 1 = \boxed{a^2 + 2ah + h^2 + 1}$

$$\frac{f(a+h)-f(a)}{h} = \frac{a^2+2ah+h^2+1-(a^2+1)}{h}$$

$$= \frac{\cancel{a^2}+2ah+h^2+\cancel{1}-\cancel{a^2}-\cancel{1}}{h}$$

$$= \frac{2ah+h^2}{h}$$

$$= \boxed{2a+h}$$

7

**Example: Find the domain.**

(a) $f(x)=\frac{1}{3x-6}$ (b) $f(x)=\sqrt[4]{x+9}^4 \geq 0^4$

$3x-6 \neq 0$

$+6 \quad +6$

$3x \neq 6$

$\boxed{x \neq 2}$

$x+9 \geq 0$

$-9 \quad -9$

$x \geq -9$

$\boxed{[-9, \infty)}$

8

**Homework**

Section#2.1: pg#155

11-17odd, 21-29odd, 37-49odd

Pre-calculus

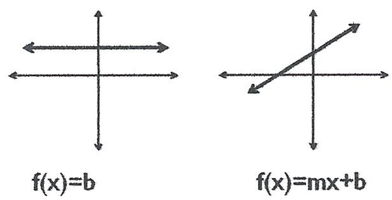
1

## Section#2.2
## Graphs of Functions

2

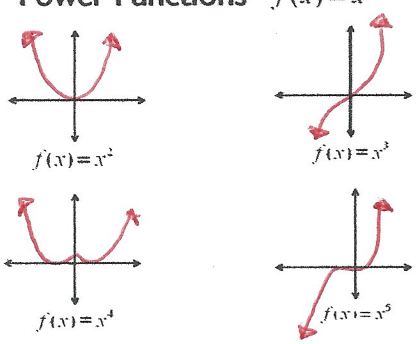
## Functions and their graphs

- Linear Functions
  f(x)=mx+b



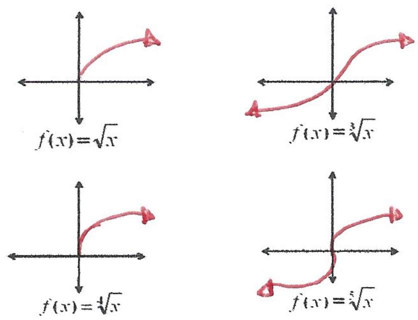

3

## Power Functions $f(x)=x^n$



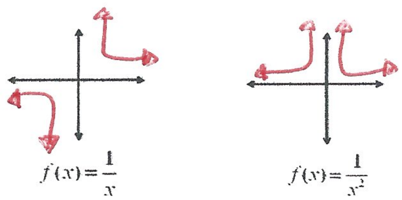

4

## Root Functions $f(x)=\sqrt[n]{x}$




5

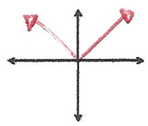
## Reciprocal Functions $f(x)=\frac{1}{x^n}$




6

## Absolute Value Function

$f(x)=|x|$

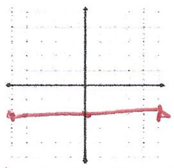


7

## Example: Graph(make a table). Find the domain and range.

(a) f(x)= -2

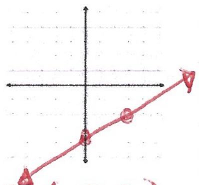
y = -2



D: (-∞,∞) R: {-2}

(b) $f(x)=\frac{2}{3}x-4$

$m=\frac{2}{3}$ b = -4

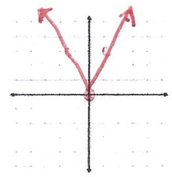


D: (-∞,∞)
R: (-∞,∞)

10

(g) $h(x)=|3x|$

| x | y |
|---|---|
| 0 | 0 |
| ±1 | 3 |
| ±2 | 6 |

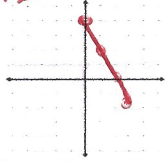


D: (-∞,∞)
R: [0,∞)

8

(c) $f(x)=-2x+4;$ $0\le x\le 3$

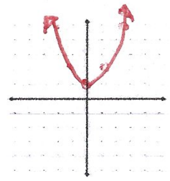
| x | y |
|---|---|
| 0 | 4 |
| 1 | 2 |
| 2 | 0 |
| 3 | -2 |



D: [0,3] R: [-2,4]

(d) $f(x)=x^2+1$

| x | y |
|---|---|
| 0 | 1 |
| ±1 | 2 |
| ±2 | 5 |



D: (-∞,∞)
R: [1,∞)

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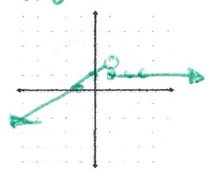
## Example: Graph the piecewise function.

(a) $f(x)=$ 1 if x _ 1; x+1 if x<1

| x | y |
|---|---|
| 1 | 2 |
| 0 | 1 |
| -1 | 0 |

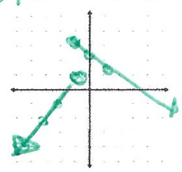
| x | y |
|---|---|
| 1 | |
| 2 | |
| 3 | |



(b) $f(x)=$ 2x+3 if x<-1; 3-x if x _ -1

| x | y |
|---|---|
| -1 | 1 |
| -2 | -1 |
| -3 | -3 |

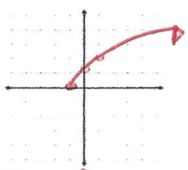
| x | y |
|---|---|
| -1 | 4 |
| 0 | 3 |
| 1 | 2 |



9

(e) $g(x)=\sqrt{x+1}$

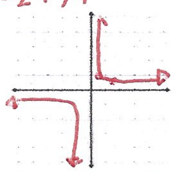
| x | y |
|---|---|
| -1 | 0 |
| 0 | 1 |
| 1 | 2 |



D: [-1,∞)
R: [0,∞)

(f) $f(x)=\frac{1}{2x}$

| x | y |
|---|---|
| 1/2 | 1 |
| 2 | 1/4 |
| -1/2 | -1 |
| -2 | -1/4 |

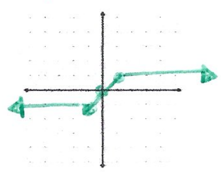


D: (-∞,0) ∪ (0,∞)
R: (-∞,0) ∪ (0,∞)

12

(c) $f(x)=\begin{cases}-1 & \text{if } x<-1\\ x & \text{if } -1\le x\le 1\\ 1 & \text{if } x>1\end{cases}$

| x | y |
|---|---|
| -1 | -1 |
| 0 | 0 |
| 1 | 1 |

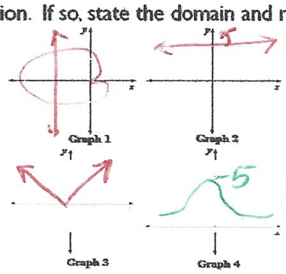


(13)

## Vertical Line Test

The curve of a graph is a function if and only if no vertical line intersects the curve more than once.

Example: Determine whether the curve is a function. If so, state the domain and range.




NO

yes
D: $(-\infty, \infty)$
R: $\{5\}$

yes
D: $(-\infty, \infty)$
R: $(0, 5]$

D: $(-\infty, \infty)$
R: $[0, \infty)$

(14)

Section#2.2: pg#167:
1-15odd, 23-31odd, 35-45odd, 55-59odd

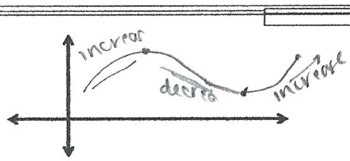
**HOMEWORK**

pre-cal

(1)

## Section#2.3
## Increasing and Decreasing Functions; Average Rate of Change

(2)



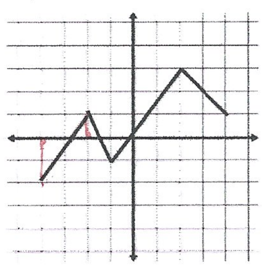

**Definition of Increasing and Decreasing Functions**

f is increasing on an interval ...
if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$

f is decreasing on an interval ...
if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$

(3)

**Example:** Determine the intervals on which the function is increasing and decreasing.



along x-axis

increasing $[-4,-2], [-1,2]$
decreasing $[-2,-1], [2,4]$

(4)

## Average Rate of Change

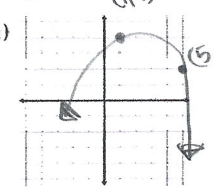
The average rate of change of the function y=f(x) between x=a and x=b

Average rate of change= $\dfrac{\text{change in y}}{\text{change in x}} = \dfrac{f(b)-f(a)}{b-a}$

(5)

**Example:** Determine the average rate of change.

(a)




$f(1)=4$
$f(5)=2$
$a=1 \quad b=5$

$$\frac{f(b)-f(a)}{b-a} = \frac{f(5)-f(1)}{5-1} = \frac{2-4}{4} = \frac{-2}{4} = \boxed{\frac{-1}{2}}$$

(6)

(b) $f(x) = 1-3x^2$; x=-2 and x=0 (a = −2, b = 0)

$$\frac{f(b)-f(a)}{b-a} = \frac{f(0)-f(-2)}{0+2}$$

$$= \frac{1+11}{2}$$

$$= \frac{12}{2} = \boxed{6}$$

$f(0)=1-3(0)^2$
$=1$

$f(-2)=1-3(-2)^2$
$=1-3(4)$
$=1-12$
$=-11$

7

(c) $f(x) = 4 - x^2$; x=1 and x=1+h

$a = 1$, $b = 1+h$

$$\frac{f(b)-f(a)}{b-a} = \frac{f(1+h)-f(1)}{1+h-1}$$

$$= \frac{\cancel{3}-2h-h^2-\cancel{3}}{h}$$

$$= \frac{-2h-h^2}{h} = \boxed{-2+h}$$

$$f(1+h) = 4-(1+h)^2$$
$$= 4-(1+2h+h^2)$$
$$= 4-1-2h-h^2 = \boxed{3-2h-h^2}$$

$$f(1) = 4-1^2$$
$$= 4-1$$
$$= \boxed{3}$$

8

## Homework

Section#2.3: pg#179
1-4all, 13-25odd, 29

Pre-calculus

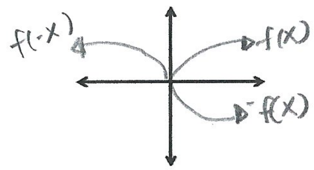
1

# Section# 2.4 Transformations of Functions

4

## Reflecting Graphs

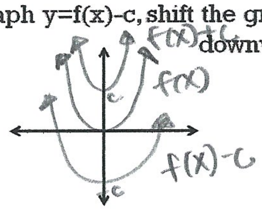
- To graph y=-f(x), reflect the graph of f(x) across the x-axis
- To graph y=f(-x), reflect the graph of f(x) across the y-axis




2

## Vertical Shifting

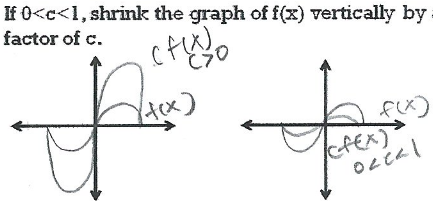
- Suppose c>o
- To graph y=f(x)+c, shift the graph of f(x) upward c units
- To graph y=f(x)-c, shift the graph of f(x) downward c units




5

## Vertical Stretching and Shrinking of Graphs

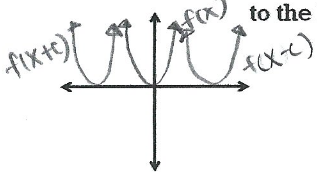
- To graph y=cf(x)
  - If c>1, stretch the graph of f(x) vertically by a factor of c.
  - If 0<c<1, shrink the graph of f(x) vertically by a factor of c.




3

## Horizontal Shifting

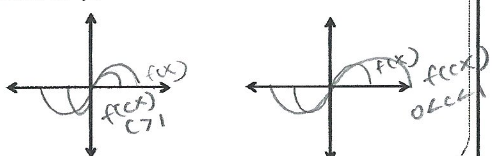
- Suppose c>0
- To graph y=f(x-c), shift the graph of f(x) to the right c units
- To graph y=f(x+c), shift the graph of f(x) to the left c units




6

## Horizontal Shrinking and Stretching of Graphs

- To graph y=f(cx)
  - If c>1, shrink the graph of f(x) vertically by a factor of $\frac{1}{c}$.
  - If 0<c<1, stretch the graph of f(x) vertically by a factor of $\frac{1}{c}$.



7

**Example:** Describe how each function can be obtained.

(a) $y=f(x+7)$
shift to the left 7

(b) $y=f(x)+7$
shift up 7

(c) $y=-2f(x)$
reflect across x-axis
vertically stretch by a factor of 2

(d) $y=3f(x)-5$
shift down 5
vertical stretch by a factor of 3

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**Example:** Explain how the graph of g is obtained.

(a) $f(x)=x^3$, $g(x)=(x-4)^3$
shift right 4

(b) $f(x)=|x|$, $g(x)=-|x+1|$
reflect across the x-axis
shift left 1

8

(e) $y=f(2x)-1$
shift down 1
horizontal shrink by a factor of $\frac{1}{2}$

(f) $y=2f(x+2)-2$
shift down 2
shift left 2
vertical stretch by a factor of 2

11

**Example:** Write the equation for the final transformed graph.

(a) $f(x)=x^3$, shift downward 1 unit and shift 4 units to the left.

$g(x)=(x+4)^3-1$

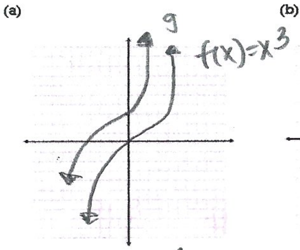
(b) $f(x)=\sqrt[3]{x}$, reflect in the y-axis, shrink vertically by a factor of $\frac{1}{5}$ and shift upward 3 units.

$g(x)=\frac{1}{5}\sqrt[3]{-x}+3$

9

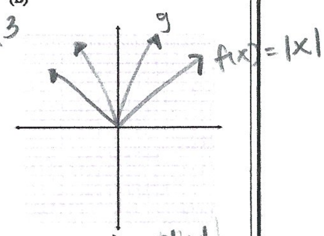
**Example:** Find the formula for the function g.

(a)




$g(x)=x^3+3$

(b)

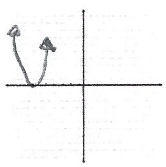



$g(x)=2|x|$

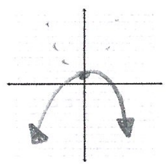
12

**Example:** Sketch the graph of the function.

(a) $f(x)=(x+7)^2$
shift left 7



(b) $f(x)=1-x^2$
$-x^2+1$
up 1
reflect x-axis

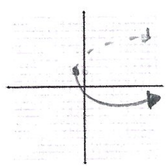


13

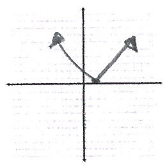
(c) $y = 2-\sqrt{x+1}$

$-\sqrt{x+1}\ +2$ up 2

left 1 reflect x axis



(d) $y = |x-1|$ right 1



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## Homework

Section#2.4: pg#

14

## Even and Odd Functions

- Let f be a function
  - f is even if f(-x)=f(x)
  - f is odd if f(-x)=-f(x)

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## Example: Determine whether the function is even, odd or neither.

(a) $f(x) = x^{-3} = \frac{1}{x^3}$

$f(-x) = \frac{1}{(-x)^3}$

$= -\frac{1}{x^3}$

$\boxed{\text{odd}}$

(b) $f(x) = x^4 - 4x^2$

$f(-x) = (-x)^4 - 4(-x)^2$

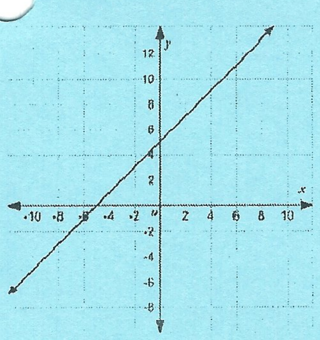
$= x^4 - 4x^2$

same

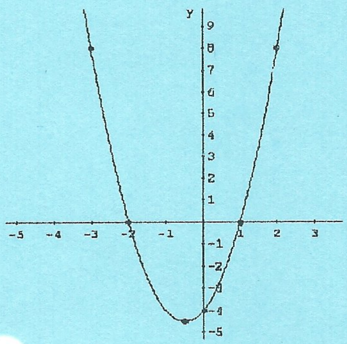
$\boxed{\text{even}}$

# The "Eight" Parent Functions

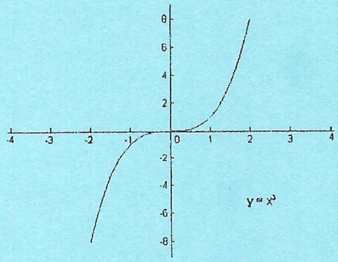
## #1) Linear Function



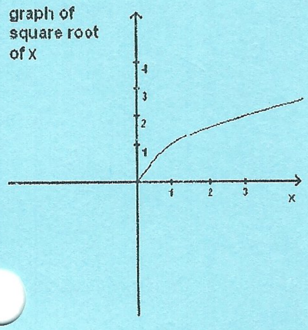
## #2) Quadratic Function



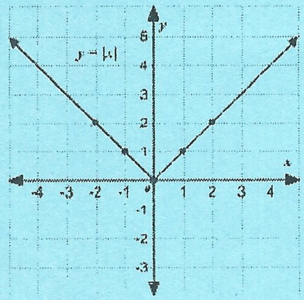
## #3) Cubic Function



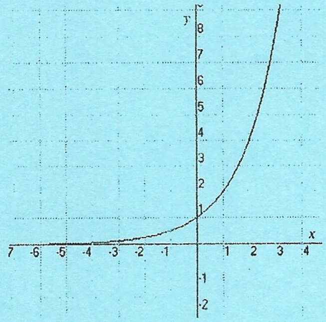

## #4) Radical(Square Root) Function



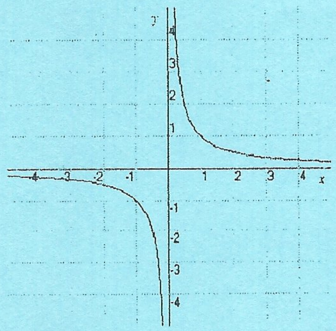

## #5) Absolute Value Function



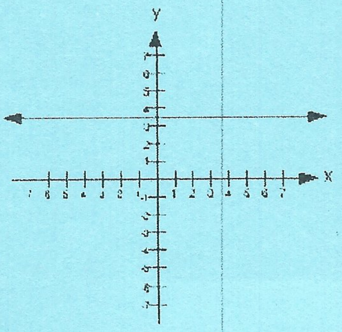

## #6) Exponential Function



## #7) Rational Function



## #8) Constant Function



Precalculus

① 

# Section#2.5
## Quadratic Functions; Maxima and Minima

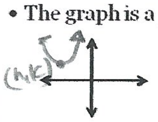
②

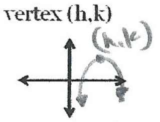
## Standard Form of a Quadratic Function

- A quadratic function $f(x)=ax^2+bx+c$ can be expressed in standard form

$$f(x)=a(x-h)^2+k$$

by competing the square
- The graph is a parabola with vertex (h,k)

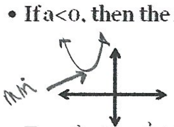


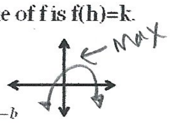




③

## Maximum or Minimum Value of a Quadratic Function

- Let $f(x)=a(x-h)^2+k$
- The maximum or minimum value occurs at x=h.
- If a>0, then the minimum value of f is f(h)=k.
- If a<0, then the maximum value of f is f(h)=k.




- For $f(x)=ax^2+bx+c$, then $x=\frac{-b}{2a}$
- For a>0, $\min = f\left(\frac{-b}{2a}\right)$
- For a<0, $\max = f\left(\frac{-b}{2a}\right)$

④

Example: Express in standard form. Find its vertex and x&y intercepts. Then graph. Find min or max.

(a) $f(x)=x^2+8x$

$=x^2+8x+16-16$

$\frac{8}{2}\to 4^2$

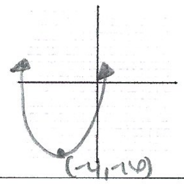
$f(x)=(x+4)^2-16$

$\boxed{\text{vertex}(-4,-16)}$

$\boxed{\min f(-4)=-16}$

x-int
$0=x^2+8x$
$x(x+8)=0$
$\boxed{x=0 \quad x=-8}$

y-int
$\boxed{y=0}$




⑤

(b) $f(x)=x^2-2x+2$

$=x^2-2x+1+2-1$

$-\frac{2}{2}\to(-1)^2$

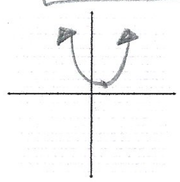
$f(x)=(x-1)^2+1$

$\boxed{\text{vertex}(1,1)}$

$\boxed{\min f(1)=1}$

x-int
$\frac{2\pm\sqrt{2^2-4(1)(2)}}{2}$
$\boxed{\text{no x-int}}$

y-int
$\boxed{y=2}$



$x=\frac{-b}{2a}=\frac{2}{2(1)}=\frac{2}{2}=1$

$y=1^2-2+2$
$y=1$
$(1,1)$

$f(x)=(x-1)^2+1$

⑥

(c) $f(x)=-x^2-4x+4$

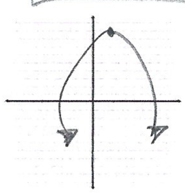
$=-(x^2+4x+4)+4+4$

$\frac{+4}{2}\to(+2)^2$

$\boxed{\max f(-2)=8}$

$(x+2)^2+8$

$\boxed{\text{vertex}(-2,8)}$



x-int
$\frac{4\pm\sqrt{16-4(-1)(4)}}{2(-1)}$

$\frac{4\pm\sqrt{32}}{-2}$

$\frac{4\pm4\sqrt{2}}{-2}$

$\boxed{x=-2\pm2\sqrt{2}}$

y-int
$\boxed{y=4}$

$x=\frac{-b}{2a}=\frac{4}{2(-1)}=-2$

$y=-4+8+4$
$y=8$
$(-2,8)$

$f(x)=-(x+2)^2+8$

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**Example: Find the maximum or minimum value.**

(a) $f(x)=10x^2+40x+113$

$a=10,\ b=40\ \ c=113$

$x=\frac{-b}{2a}=\frac{-40}{2(10)}=\frac{-40}{20}=-2$

$f(-2)=10(-2)^2+40(-2)+113$

$=40-8+113=73$

$\boxed{\min f(-2)=73}$

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(b) $f(x)=1+3x-x^2$

$a=-1\ \ b=3\ \ c=1$

$x=\frac{-b}{2a}=\frac{-3}{2(-1)}=\frac{3}{2}$

$f(\frac{3}{2})=1+3(\frac{3}{2})-(\frac{3}{2})^2$

$=1+\frac{9}{2}-\frac{9}{4}=\frac{4}{4}+\frac{18}{4}-\frac{9}{4}=\frac{13}{4}$

$\boxed{\max f(\frac{3}{2})=\frac{13}{4}}$

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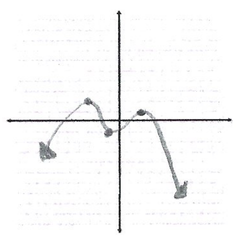
(c)



$\boxed{\begin{array}{l}\max=2,1\\ \min=-1\end{array}}$

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# Homework

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PG=

example

(a) $f(x) = x^2 + 8x$ $a=1$ $b=8$ $c=0$

$x = \frac{-b}{2a} = \frac{-8}{2(1)} = \frac{-8}{2} = \boxed{-4}$ $\{x=-4\}$

$y = (-4)^2 + 8(-4) = 16 - 32 = \boxed{-16}$ $\{y=-16\}$

$f(x) = (x + +4)^2 - 16$

$\{f(x) = (x+4)^2 - 16\}$ $\boxed{\text{vertex } (-4,-16)}$

$\boxed{\min f(-4) = -16}$

x-int let $y=0$

$x^2 + 8x = 0$

$x(x+8) = 0$

$x=0$ $x+8=0$

$\boxed{x=0 \quad x=-8}$

y-int let $x=0$

$y = 0^2 + 8(0)$

$\boxed{y=0}$

pre-calculus

①

# Section#2.7
## Combining Functions

②

## Algebra of Functions

- Let f and g be functions with domains A and B

(f+g)=f(x) + g(x) Domain: $A \cap B$
(f-g)=f(x) - g(x) Domain: $A \cap B$
(fg)(x)=f(x)•g(x) Domain: $A \cap B$
$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$ Domain: $A \cap B$ and g(x)≠0

③

## Composition of Functions

- Given two functions f and g, the composition of f and g is defined by

$$(f \circ g)(x) = f(g(x))$$

④

**Example: Find f+g, f-g, fg, and f/g and their domains.**

(a) $f(x)=\frac{2}{x+1}, g(x)=\frac{x}{x+1}$

$f+g = \frac{2}{x+1} + \frac{x}{x+1} = \boxed{\frac{x+2}{x+1}}$ D: $x+1=0$, $x \neq 1$

$f-g = \frac{2}{x+1} - \frac{x}{x+1} = \boxed{\frac{2-x}{x+1}}$ D: $x \neq 1$

$fg = \frac{2}{x+1} \cdot \frac{x}{x+1} = \boxed{\frac{2x}{(x+1)^2}}$ D: $x \neq 1$

$\frac{f}{g} = \frac{\frac{2}{x+1}}{\frac{x}{x+1}} = \frac{2}{\cancel{x+1}} \cdot \frac{\cancel{x+1}}{x} = \boxed{\frac{2}{x}}$ D: $x \neq 1$, $x \neq 0$

⑤

(b) $f(x)=\sqrt{9-x^2}, g(x)=\sqrt{x^2-4}$

$f+g = \boxed{\sqrt{9-x^2}+\sqrt{x^2-4}}$ D: $[-3,-2] \cup [2,3]$

$f-g = \boxed{\sqrt{9-x^2}-\sqrt{x^2+4}}$ D: $[-3,-2] \cup [2,3]$

$fg = \sqrt{9-x^2} \cdot \sqrt{x^2-4}$ = foil

$= \boxed{\sqrt{-x^4+13x^2-36}}$ D: $[-3,-2] \cup [2,3]$

$\frac{f}{g} = \boxed{\frac{\sqrt{9-x^2}}{\sqrt{x^2-4}}}$ D: $[-3,-2) \cup (2,3]$

cant equal -2 or 2

D: $\sqrt{9-x^2} \geq 0$
$9-x^2 \geq 0$
$(3-x)(3+x) \geq 0$
− + −
-3 3

$\sqrt{x^2-4} \geq 0$
$(x+2)(x-2) \geq 0$
+ −
-2 2

$[-3,2] \cup [2,3]$

⑥

**Example: Let $f(x)=3x-5$ and $g(x)=2-x^2$ to evaluate.**

(a) $f(f(4))$

$f(4) = 3(4)-5$
$= 12-5$
$= 7$
$f(7) = 3(7)-5$
$= 21-5$
$= \boxed{16}$

(b) $(g \circ f)(-2)$

$g(f(-2))$
$f(-2) = 3(-2)-5$
$= -6-5$
$= -11$
$g(-11) = 2-(-11)^2$
$= 2-121$
$= \boxed{-119}$

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(c) $(f \circ g)(x)$

$f(g(x))$

$g(x) = 2 - x^2$

$f(2-x^2) = 3(2-x^2) - 5$

$6 - 3x^2 - 5$

$= \boxed{-3x^2+1}$

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**Homework**

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1-9odd, 17-21odd,
29-33odd, 41, 43

8

**Example: Find** $f \circ g, g \circ f, f \circ f, g \circ g$
~~and their domains.~~

$f(x) = x^2, g(x) = \sqrt{x-3}$

$f \circ g = f(g(x)) = f(\sqrt{x-3}) = (\sqrt{x-3})^2 = \boxed{x-3}$

$g \circ f = g(f(x)) = g(x^2) = \boxed{\sqrt{x^2-3}}$

$f \circ f = f(f(x)) = f(x^2) = (x^2)^2 = \boxed{x^4}$

$g \circ g = g(g(x)) = g(\sqrt{x-3}) = \boxed{\sqrt{\sqrt{x-3} - 3}}$

$g(g(x))$
$\downarrow$
$g(\sqrt{x-3}) = \sqrt{\sqrt{x-3} - 3}$

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**Example: Find** $f \circ g \circ h$

$f(x) = \frac{1}{x}, g(x) = x^3, h(x) = x^2 + 2$

$f(g(h(x)))$

$f(g(x^2+2))$

$f((x^2+2)^3)$

$\boxed{\dfrac{1}{(x^2+2)^3}}$

pre-calculus

1

## Section#2.8
## One-to-one Functions and their Inverses

2

## Definition of One-to-One

A function with domain A is called a one-to-one function if no two elements of A have the same image

whenever $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$

Ex:

| one-to-one | NOT one-to-one |
|---|---|
| $1 \Rightarrow 5$ | $1 \Rightarrow 5$ |
| $2 \Rightarrow 10$ | $2 \Rightarrow 10$ |
| $3 \Rightarrow 15$ | $3 \Rightarrow 10$ |

3

## Horizontal Line Test

A function is one-to-one if and only if no horizontal lines intersect its graph more than once.

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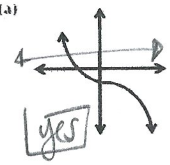
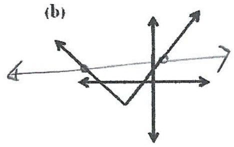
Example: Determine if the function is one-to-one?

(a)



$\boxed{\text{yes}}$

(b)



$\boxed{\text{no}}$

(c) f(x)=3x-2

y=mx+b

$\boxed{\text{yes}}$

(d) $f(x)=|x|$

$\boxed{\text{no}}$

5

## Definition of the Inverse of a Function

Let f be a one-to-one function with domain A and range B. The its inverse $f^{-1}$ has domain B and range A, defined by

$$f^{-1}(y) = x \Leftrightarrow f(x) = y$$

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## Example:

(a) If f(5)=7, then $f^{-1}(7)=$ 5

(b) If $f^{-1}(4)=2$ , then f(2)= 4

7

## Inverse Property

Let f be one-to-one and $f^{-1}$ be the inverse.

$$f^{-1}(f(x)) = x$$

$$f(f^{-1}(x)) = x$$

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## Example: Show that f and g are inverses.

(a) $f(x) = 3x,\ g(x) = \frac{x}{3}$

$f(g(x)) = f\left(\frac{x}{3}\right)$

$= \cancel{3}\left(\frac{x}{\cancel{3}}\right)$

$= \boxed{x}$

(b) $f(x) = \frac{3-x}{4},\ g(x) = 3-4x$

$f(g(x))$

$f(3-4x)$

$= \frac{3-(3-4x)}{4}$

$= \frac{\cancel{3}-\cancel{3}+4x}{4}$

$= \frac{4x}{4} = \boxed{x}$

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## Finding the inverse
1. Switch x and y
2. Solve for y

Example: Find the inverse.

(a) f(x)=3-5x

$y = 3-5x$

$x = 3-5y$

$-3 \quad -3$

$\frac{x-3}{5} = \frac{-5y}{5}$

$-\frac{x+3}{5} = y$

$\boxed{f^{-1}(x) = \frac{-x+3}{5}}$

(b) $f(x) = \frac{x-2}{x+2}$

$y = \frac{x-2}{x+2}$

$\frac{x}{1} = \frac{y-2}{y+2}$

$xy + 2x = y - 2$

$-y \ {-2x} \quad -y \ {-2x}$

$xy - y = -2x - 2$

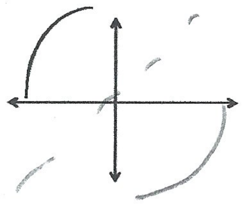
$\frac{y(x-1)}{x-1} = \frac{-2x-2}{x-1}$

$y = \boxed{\frac{-2x-2}{x-1} = f^{-1}(x)}$

10

## Graphing the inverse
reflect across the line y=x

Example: Graph the inverse.



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## Homework

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