Exam

Name Answer key (1)

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the radius of convergence of the power series.

1) $\sum_{n=0}^{\infty} (x-4)^n$ 1) B

A) ∞ B) 1 C) 0 D) 2

2) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{9n+4}$ 2) D

A) 2 B) 0 C) ∞ D) 1

Find the interval of convergence of the series.

3) $\sum_{n=0}^{\infty} \frac{(x-2)^{2n}}{4^n}$ 3) C

A) $1 < x < 3$ B) $x < 4$ C) $0 < x < 4$ D) $-4 < x < 4$

4) $\sum_{n=0}^{\infty} (x-8)^n$ 4) B

A) $7 \le x < 9$ B) $7 < x < 9$ C) $x < 9$ D) $-9 < x < 9$

Solve the problem.

5) Find the coefficient of x^3 in the Maclaurin series generated by $f(x) = e^{-3x}$. 5) D

A) -3 B) -27 C) $\frac{-3}{6}$ D) $\frac{-27}{6}$

Find the Taylor polynomial of the indicated order and use it to approximate the function with the given value.

6) sin 0.195 (3) 6) B

A) 0.1760477 B) 0.1937665 C) 0.1962382 D) 0.1925848

~~**Solve the problem.**~~

~~7) The polynomial $1 + 7x + 21x^2$ is used to approximate $f(x) = (1 + x)^7$ on the interval $-0.01 \le x \le 0.01$. Use the Remainder Estimation Theorem to estimate the maximum absolute error.~~ 7) ~~A~~

~~A) $\approx 3.642 \times 10^{-5}$ B) $\approx 2.061 \times 10^{-5}$ C) $\approx 3.642 \times 10^{-4}$~~

Approximate the given function by using the third order Taylor polynomial.

8) ln 0.78 8) D

A) -0.4557 B) 0.7523 C) -0.3007 D) -0.2477

Find the Taylor polynomial of third order for the function at $x = 0$.

9) $f(x) = \frac{1}{x + 4}$

A) $\frac{1}{4} - \frac{x}{16} + \frac{x^2}{64} - \frac{x^3}{256}$

B) $\frac{x}{4} - \frac{x^2}{16} + \frac{x^3}{64} - \frac{x^4}{256}$

C) $\frac{x}{4} + \frac{x^2}{16} + \frac{x^3}{64} + \frac{x^4}{256}$

D) $\frac{1}{4} + \frac{x}{16} + \frac{x^2}{64} + \frac{x^3}{256}$

9) A

Find the first four nonzero terms of the Maclaurin expansion of the given function.

10) $f(x) = \cos x^5$

A) $x + 5x + \frac{5x^2}{2!} + \frac{5x^3}{3!} + \ldots$

B) $1 + 5x + \frac{5x^2}{2!} + \frac{5x^3}{3!} + \ldots$

C) $1 - \frac{x^{10}}{2!} + \frac{x^{20}}{4!} - \frac{x^{30}}{6!} + \ldots$

D) $1 - x^5 + \frac{2x^{10}}{3} - \frac{x^{20}}{120} + \ldots$

10) C

Find the general term of the Maclaurin series for the given function.

11) e^{8x}

A) $\sum_{n=1}^{\infty} \frac{(-1)^n 8^n x^n}{n!}$

B) $\sum_{n=0}^{\infty} \frac{8^n x^n}{n!}$

C) $\sum_{n=0}^{\infty} \frac{(-1)^n 8^n x^n}{n!}$

D) $\sum_{n=1}^{\infty} \frac{8^n x^n}{n!}$

11) B

Answer Key
Testname: POWER SERIES REVIEW

1) B
2) D
3) C
4) B
5) D
6) B
7) A
8) D
9) A
10) C
11) B

① $\lim_{n\to\infty} \left| \frac{(x-4)^{n+1}}{(x-4)^n} \right| = \lim_{n\to\infty} \left| \frac{\cancel{(x-4)^n}(x-4)}{\cancel{(x-4)^n}} \right| = |x-4| < 1$ radius

② $\lim_{n\to\infty} \left| \frac{(x-2)^{n+1}}{9(n+1)+4} \cdot \frac{9n+4}{(x-2)^n} \right| = \lim_{n\to\infty} \left| \frac{\cancel{(x-2)^n}(x-2)}{9n+13} \cdot \frac{9n+4}{\cancel{(x-2)^n}} \right|$

$9n+9+4$

$= \lim_{n\to\infty} \left| \frac{9n+4}{9n+13} \cdot (x-2) \right| = 1 \cdot |x-2| < 1$

$|x-2| < 1$ radius

③ $\lim_{n\to\infty} \left| \frac{(x-2)^{2(n+1)}}{4^{n+1}} \cdot \frac{4^n}{(x-2)^{2n}} \right| = \lim_{n\to\infty} \left| \frac{\cancel{(x-2)^{2n}}(x-2)^2}{\cancel{4^n} \cdot 4^1} \cdot \frac{\cancel{4^n}}{\cancel{(x-2)^{2n}}} \right|$

$= \lim_{n\to\infty} \left| \frac{(x-2)^2}{4} \right| = \left| \frac{(x-2)^2}{4} \right| < 1$

$\sqrt{(x-2)^2} < \sqrt{4}$

$|x-2| < 2$

$-2 < x-2 < 2$
$+2 \quad\quad +2 \quad +2$

$0 < x < 4$

try 0: $\frac{(0-2)^{2n}}{4^n} = \frac{((-2)^2)^n}{4^n} = \frac{4^n}{4^n} = 1$

$1+1+1$ divegi

try 4: $\frac{(4-2)^{2n}}{4^n} = \frac{(2^2)^n}{4^n} = \frac{4^n}{4^n} = 1$

$1+1+\ldots$ divegse

④ $\lim_{n\to\infty}\left|\frac{(x-8)^{n+1}}{(x-8)^n}\right| = \lim_{n\to\infty}\left|\frac{\cancel{(x-8)^n}(x-8)^1}{\cancel{(x-8)^n}}\right| = |x-8| < 1 \ldots$

$$-1 < x-8 < 1$$
$$+8 \quad +8 \quad +8$$
$$\boxed{7 < x < 9}$$

try 7: $(7-8)^n = (-1)^n = -1+1-1+$ Divergic

try 9: $(9-8)^n = 1^n = 1+1+1 =$ Divergic

⑤ e^{-3x} $\qquad e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

$$e^{-3x} = 1 + (-3x) + \frac{(-3x)^2}{2!} + \frac{(-3x)^3}{3!}$$

$$1 - 3x + \frac{9x^2}{2!} - \frac{27x^3}{3!} \qquad -\frac{27}{6}$$

⑥ $\sin(0.195)$ order 3

$$x - \frac{x^3}{3!} = (0.195) - \frac{(0.195)^3}{3!} = \boxed{0.193764} \quad B$$

⑦

$$R_n(x) = \frac{f^{n+1}(z)(x-c)^{n+1}}{(n+1)!}$$

$n = 2$

$c = 0$

$x = 0.01$

$z =$

$$R_2(x) = \frac{f^3(z)(x-0)^3}{3!}$$

$$R_2(x) = f^3(z)\frac{x^3}{3!}$$

n	
0	$(1+x)^7$
1	$7(1+x)^6$
2	$42(1+x)^5$
3	$210(1+x)^4$
4	$\cancel{840(1+x)^3}$

$$R_2(x) = 210(1+z)^4\frac{x^3}{3!}$$

$$= 210(1+.01)^4\frac{(.01)^3}{3!}$$

$$= .00003642$$

$$\boxed{3.642 \times 10^{-5}}$$

⑧ $\ln(0.78) =$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3}$$

$$\ln(1+(-.22)) = (-.22) - \frac{(-.22)^2}{2} + \frac{(-.22)^3}{3} = \boxed{-.240651}$$

⑨

n	derivative of $\frac{1}{x+4}$	plug in 0	n!	$(x-0)^n$
0	$\frac{1}{x+4}$ $(x+4)^{-1}$	$\frac{1}{4}$	1	1
1	$\frac{-1}{(x+4)^2}$	$\frac{-1}{16}$	1	x
2	$\frac{2}{(x+4)^3}$ $\frac{2}{64}$	$\frac{1}{32}$	2!	x^2
3	$\frac{-6}{(x+4)^4}$ $\frac{-6}{4}$	$\frac{-3}{128}$	3!	x^3

$$\frac{1}{4} - \frac{x}{16} + \frac{x^2}{64} - \frac{x^3}{256}$$

(10) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$

$$\cos(x^5) = 1 - \frac{(x^5)^2}{2!} + \frac{(x^5)^4}{4!} - \frac{(x^5)^6}{6!}$$

$$1 - \frac{x^{10}}{2!} + \frac{x^{20}}{4!} - \frac{x^{30}}{6!}$$

(11)

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

$$e^{8x} = 1 + (8x) + \frac{(8x)^2}{2!} + \frac{(8x)^3}{3!} + \frac{(8x)^4}{4!}$$

$$1 + 8x + \frac{8^2x^2}{2!} + \frac{8^3x^3}{3!} + \frac{8^4x^4}{4!}$$

$$\frac{8^n x^n}{n!}$$