Ex # 1 Find the nth Maclaurin polynomial for $f(x) = e^x$. $c=0$

n	$f^n(x)$ (derivative)	$f^n(c)$ $c=0$ (derivative at c)	$(x-c)^n$	$n!$	$\frac{f^n(c)(x-c)^n}{n!}$
0	e^x	$e^0=1$	$(x-0)^0 = 1$	$0!=1$	$\frac{1\cdot 1}{1}$
1	e^x	$e^0=1$	$(x-0)^1 = x$	$1!$	$\frac{1\cdot x}{1!}$
2	e^x	$e^0=1$	$(x-0)^2 = x^2$	$2!$	$\frac{1\cdot x^2}{2!}$
3	e^x	$e^0=1$	$(x-0)^3 = x^3$	$3!$	$\frac{1\cdot x^3}{3!}$
4	e^x	$e^0=1$	$(x-0)^4 = x^4$	$4!$	$\frac{1\cdot x^4}{4!}$
5	e^x	$e^0=1$	$(x-0)^5 = x^5$	$5!$	$\frac{1\cdot x^5}{5!}$

$$1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}+\dots \quad \sum_{n=0}^{\infty}\frac{x^n}{n!}$$

Ex # 2 Find the Taylor polynomials P_0, P_1, P_2, P_3, and P_4 for $f(x) = \frac{1}{1-x}$ $c=0$

n	$f^n(x)$ (derivative)	$f^n(c)$ (derivative at c)	$(x-c)^n$	$n!$	$\frac{f^n(c)(x-c)^n}{n!}$
0	$\frac{1}{1-x} = (1-x)^{-1}$	$\frac{1}{1-0}=1$	$(x-0)^0 = 1$	$0!=1$	$\frac{1\cdot 1}{1}$
1	$-1(1-x)^{-2}\cdot -1 = \frac{1}{(1-x)^2}$	$\frac{1}{(1-0)^2}=1$	$(x-0)^1 = x$	$1!$	$\frac{1\cdot x}{1!}$
2	$-2(1-x)^{-3}\cdot -1 = \frac{2}{(1-x)^3}$	$\frac{2}{(1-0)^3}=2$	$(x-0)^2 = x^2$	$2!$	$\frac{2x^2}{2!}$
3	$-6(1-x)^{-4}\cdot -1 = \frac{6}{(1-x)^4}$	$\frac{6}{(1-0)^4}=6$	$(x-0)^3 = x^3$	$3!$	$\frac{6x^3}{3!}$
4	$-24(1-x)^{-5}\cdot -1$ $\frac{24}{(1-x)^5}$	$\frac{24}{(1-0)^5}=24$	$(x-0)^4 = x^4$	$4!$	$\frac{24x^4}{4!}$

$$1+\frac{x}{1!}+\frac{2x^2}{2!}+\frac{6x^3}{3!}+\frac{24x^4}{4!}+\dots = 1+x+x^2+x^3+x^4+\dots \sum_{n=0}^{\infty} x^n$$

$1\cdot 2 \quad 1\cdot 2\cdot 3 \quad 1\cdot 2\cdot 3\cdot 4$

Ex # 3 Find the Maclaurin polynomial P_6 for $f(x) = \cos x$. $c=0$

n	$f^n(x)$ (derivative)	$f^n(c)$ (derivative at c)	$(x-c)^n$	$n!$	$\frac{f^n(c)(x-c)^n}{n!}$
0	$\cos x$	$\cos(0) = 1$	$(x-0)^0 = 1$	$0!=1$	$\frac{1\cdot 1}{1} = 1$
1	$-\sin x$	$-\sin(0) = 0$	$(x-0)^1 = x$	$1!$	$\frac{0\cdot x}{1!} = 0$
2	$-\cos x$	$-\cos(0) = -1$	$(x-0)^2 = x^2$	$2!$	$\frac{-1\cdot x^2}{2!}$ $-x^2/2!$
3	$\sin x$	$\sin(0) = 0$	$(x-0)^3 = x^3$	$3!$	$\frac{0\cdot x^3}{3!} = 0$
4	$\cos x$	$\cos(0) = 1$	$(x-0)^4 = x^4$	$4!$	$1\cdot x^4/4!$ $x^4/4!$
5	$-\sin x$	$-\sin(0) = 0$	$(x-0)^5 = x^5$	$5!$	$0\cdot x^5/5! = 0$
6	$-\cos x$	$-\cos(0) = -1$	$(x-0)^6 = x^6$	$6!$	$-1\cdot x^6/6! =$ $-x^6/6$

$$1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\dots \quad \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$$

x # 1 Find the nth Maclaurin polynomial for $f(x) = \sin x$ $c=0$

n	$f^n(x)$ (derivative)	$f^n(c)$ (derivative at c)	$(x-c)^n$	$n!$	$\frac{f^n(c)(x-c)^n}{n!}$
0	$\sin x$	$\sin(0)=0$	$(x-0)^0=1$	$0!=1$	$\frac{0\cdot 1}{1}=0$
1	$\cos x$	$\cos(0)=1$	$(x-0)^1=x$	$1!$	$1\cdot x/1!$
2	$-\sin x$	$\sin(0)=0$	$(x-0)^2=x^2$	$2!$	$0\cdot x^2/2!=0$
3	$-\cos x$	$\cos(0)=-1$	$(x-0)^3=x^3$	$3!$	$-1\cdot x^3/3!$
4	$\sin x$	$\sin(0)=0$	$(x-0)^4=x^4$	$4!$	$0\cdot x^4/4!=0$
5	$\cos x$	$\cos(0)=1$	$(x-0)^5=x^5$	$5!$	$1\cdot x^5/5!$

$$\frac{x}{1!}-\frac{x^3}{3!}+\frac{x^5}{5!}+\dots \quad \sum_{n=0} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

x # 2 Find the Taylor polynomials $P_0, P_1, P_2, P_3,$ and P_4 for $f(x) = \ln x$ at $x=1$

n	$f^n(x)$ (derivative)	$f^n(c)$ (derivative at c)	$(x-c)^n$	$n!$	$\frac{f^n(c)(x-c)^n}{n!}$
0	$\ln x$	$\ln(1)=0$	$(x-1)^0=1$	$0!=1$	$0\cdot 1/1=0$
1	$\frac{1}{x}$ x^{-1}	$\frac{1}{1}=1$	$(x-1)^1$	$1!$	$1(x-1)/1!$
2	$-1x^{-2}=-\frac{1}{x^2}$	$-\frac{1}{1^2}=-1$	$(x-1)^2$	$2!$	$-1(x-1)^2/2!$
3	$2x^{-3}$ $\frac{2}{x^3}$	$\frac{2}{1^3}=2$	$(x-1)^3$	$3!$	$2(x-1)^3/3!$
4	$-6x^{-4}$ $-6/x^4$	$\frac{-6}{1^4}=-6$	$(x-1)^4$	$4!$	$-6(x-1)^4/4!$

$$\frac{(x-1)}{1!}-\frac{(x-1)^2}{\underset{1\cdot 2}{2!}}+\frac{2(x-1)^3}{\underset{1\cdot 2\cdot 3}{3!}}-\frac{6(x-1)^4}{\underset{1\cdot 2\cdot 3\cdot 4}{4!}}+$$

$$\frac{(x-1)}{1}-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}-\frac{(x-1)^4}{4}+\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n}$$

x # 3 Find the Maclaurin polynomial P_6 for $f(x) = \cos(2x)$ $c=0$

n	$f^n(x)$ (derivative)	$f^n(c)$ (derivative at c)	$(x-c)^n$	$n!$	$\frac{f^n(c)(x-c)^n}{n!}$
0	$\cos(2x)$	$\cos(0)=1$	$(x-0)^0=1$	$0!=1$	$1\cdot 1/1=1$
1	$-\sin(2x)\cdot 2=-2\sin(2x)$	$-2\sin(0)=0$	$(x-0)^1=x$	$1!$	$0\cdot x/1!=0$
2	$-2\cos(2x)\cdot 2=-4\cos(2x)$	$-4\cos(0)=-4$	$(x-0)^2=x^2$	$2!$	$-4x^2/2!$
3	$4\sin(2x)\cdot 2=8\sin(2x)$	$8\sin(0)=0$	$(x-0)^3=x^3$	$3!$	$0\,x^3/3!=0$
4	$8\cos(2x)\cdot 2=16\cos(2x)$	$16\cos(0)=16$	$(x-0)^4=x^4$	$4!$	$16\cdot x^4/4!$
5	$-16\cos(2x)\cdot 2=-32\sin(2x)$	$-32\sin(0)=0$	$(x-0)^5=x^5$	$5!$	$0x^5/5!=0$
6	$-32\cos(2x)\cdot 2=-64\cos(2x)$	$-64\cos(0)=-64$	$(x-0)^6=x^6$	$6!$	$-64x^6/6!$

$$1-\frac{4x^2}{2!}+\frac{16x^4}{4!}-\frac{64x^6}{6!}+\dots \quad \sum_{n=0}^{\infty}\frac{(-1)^n 2^{2n}x^{2n}}{(2n)!}$$

Name Answer key

power series Homework

period # ____

16. The coefficient of x^4 in the Maclaurin series for $f(x) = e^{-x/2}$ is

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

(A) $-\frac{1}{24}$ (B) $\frac{1}{24}$ (C) $\frac{1}{96}$ (D) $-\frac{1}{384}$ (E) $\frac{1}{384}$

$$1 + \left(-\frac{x}{2}\right) + \frac{\left(-\frac{x}{2}\right)^2}{2!} + \frac{\left(-\frac{x}{2}\right)^3}{3!} + \frac{\left(-\frac{x}{2}\right)^4}{4!} = \frac{\frac{x^4}{16}}{4!} = \frac{x^4}{16}\cdot\frac{1}{4!} = \frac{x^4}{16\cdot 4\cdot 3\cdot 2\cdot 1} = \frac{1}{384}x^4$$

30. The Taylor polynomial of order 3 at $x = 0$ for $f(x) = \sqrt{1+x}$ is

(A) $1 + \frac{x}{2} - \frac{x^2}{4} + \frac{3x^3}{8}$ (B) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$

(C) $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16}$ (D) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{8}$

(E) $1 - \frac{x}{2} + \frac{x^2}{4} - \frac{3x^3}{8}$

n	$\frac{d}{dx}$	plug in 0	n!	$(x-c)^n$
0	$(1+x)^{1/2}$	1	1	1
1	$\frac{1}{2}(1+x)^{-1/2}$	$\frac{1}{2}$	1!	x
2	$-\frac{1}{4}(1+x)^{-3/2}$	$-\frac{1}{4}$	2!	x^2
3	$\frac{3}{8}(1+x)^{-5/2}$	$\frac{3}{8}$	3!	x^3

$1 + \frac{1}{2}x - \frac{1}{4\cdot 2!}x^2 + \frac{3}{8\cdot 3!}x^3 +$ $\quad 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$

31. The Taylor polynomial of order 3 at $x = 1$ for e^x is

(A) $1 + (x-1) + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$

(B) $e\left[1 + (x-1) + \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}\right]$

(C) $e\left[1 + (x+1) + \frac{(x+1)^2}{2!} + \frac{(x+1)^3}{3!}\right]$

(D) $e\left[1 + (x-1) + \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3!}\right]$

(E) $e\left[1 - (x-1) + \frac{(x-1)^2}{2!} + \frac{(x-1)^3}{3!}\right]$

n	$\frac{d}{dx}$	plug in 1	n!	fractio	$(x-c)^n$
0	e^x	e^1	1	e	1
1	e^x	e	1!	e	$(x-1)$
2	e^x	e	2!	$e/2!$	$(x-1)^2$
3	e^x	e	3!	$e/3!$	$(x-1)^3$

$$e + e(x-1) + \frac{e(x-1)^2}{2!} + \frac{e(x-1)^3}{3!} + \ldots$$

35. The coefficient of x^2 in the Maclaurin series for $e^{\sin x}$ is

$e^x = 1 + x + \frac{x^2}{2!}$

(A) 0 (B) 1 (C) $\frac{1}{2!}$

(D) -1 (E) $\frac{1}{4}$

$e^{\sin x} = 1 + (\sin x) + \frac{(\sin x)^2}{2!}$

37. The coefficient of $(x-1)^5$ in the Taylor series for $x \ln x$ about $x = 1$ is

(A) $-\frac{1}{20}$ (B) $\frac{1}{5!}$ (C) $-\frac{1}{5!}$ (D) $\frac{1}{4!}$ (E) $-\frac{1}{4!}$

44. The Taylor polynomial of order 3 at $x = 0$ for $(1+x)^p$, where p is a constant, is

(A) $1 + px + p(p-1)x^2 + p(p-1)(p-2)x^3$

(B) $1 + px + \frac{p(p-1)}{2}x^2 + \frac{p(p-1)(p-2)}{3}x^3$

(C) $1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3$

(D) $px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3$

(E) none of these

n	$\frac{d}{dx}$	plug in 0	$n!$	$(x-c)^n$
0	$(1+x)^p$	1	1	1
1	$p(1+x)^{p-1}$	p	$1!$	x
2	$p(p-1)(1+x)^{p-2}$	$p(p-1)$	$2!$	x^2
3	$p(p-1)(p-2)(1+x)^{p-3}$	$p(p-1)(p-2)$	$3!$	x^3

$1 + px + \frac{p(p-1)x^2}{2!} + \frac{p(p-1)(p-2)x^3}{3!}$

45. The Taylor series for $\ln(1+2x)$ about $x = 0$ is

(A) $2x - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} - \frac{(2x)^4}{4} + \cdots$

(B) $2x - 2x^2 + 8x^3 - 16x^4 + \cdots$

(C) $2x - 4x^2 + 16x^3 + \cdots$

(D) $2x - x^2 + \frac{8}{3}x^3 - 4x^4 + \cdots$

(E) $2x - \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} - \frac{(2x)^4}{4!} + \cdots$

$2x - 2x^2 + \frac{8x^3}{3}$

n	$\frac{d}{dx}$	plug in 0	$n!$	$(x-c)^n$
0	$\ln(1+2x)$	0	1	1
1	$\frac{1}{1+2x}\cdot 2 = \frac{2}{1+2x}$; $2(1+2x)^{-1}$	2	$1!$	x
2	$-2(1+2x)^{-2}\cdot 2 = -4(1+2x)^{-2}$	-4	$2!$	x^2
3	$8(1+2x)^{-3}\cdot 2$	16	$3!$	x^3

$2x - \frac{4}{2!}x^2 = 2x - 2x^2 + \frac{16x^3}{3!}$

47. The third-order Taylor polynomial $P_3(x)$ for $\sin x$ about $\frac{\pi}{4}$ is

(A) $\frac{1}{\sqrt{2}}\left((x-\frac{\pi}{4}) - \frac{1}{3!}(x-\frac{\pi}{4})^3\right)$

(B) $\frac{1}{\sqrt{2}}\left(1 + (x-\frac{\pi}{4}) - \frac{1}{2}(x-\frac{\pi}{4})^2 + \frac{1}{3!}(x-\frac{\pi}{4})^3\right)$

(C) $\frac{1}{\sqrt{2}}\left(1 + (x-\frac{\pi}{4}) - \frac{1}{2!}(x-\frac{\pi}{4})^2 - \frac{1}{3!}(x-\frac{\pi}{4})^3\right)$

(D) $1 + (x-\frac{\pi}{4}) - \frac{1}{2}(x-\frac{\pi}{4})^2 - \frac{1}{6}(x-\frac{\pi}{4})^3$

(E) $\frac{1}{\sqrt{2}}\left(1 + x - \frac{x^2}{2!} - \frac{x^3}{3!}\right)$

n	$\frac{d}{dx}$	plug in $\pi/4$	$n!$	$(x-c)^n$
0	$\sin x$	$\sqrt{2}/2$	1	1
1	$\cos x$	$\sqrt{2}/2$	$1!$	$(x-\pi/4)$
2	$-\sin x$	$-\sqrt{2}/2$	$2!$	$(x-\pi/4)^2$
3	$-\cos x$	$-\sqrt{2}/2$	$3!$	$(x-\pi/4)^3$

$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}(x-\pi/4) - \frac{\sqrt{2}}{2\cdot 2!}(x-\pi/4)^2 - \frac{\sqrt{2}}{2\cdot 3!}(x-\pi/4)^3 + \ldots = \frac{\sqrt{2}}{2}\left(1 + (x-\pi/4) - \frac{1}{2!}(x-\pi/4)^2 - \frac{1}{3!}(x-\pi/4)^3\right)$

48. Let h be a function for which all derivatives exist at $x = 1$. If $h(1) = h'(1) = h''(1) = h'''(1) = 6$, which third-degree polynomial best approximates h there?

(A) $6 + 6x + 6x^2 + 6x^3$

(B) $6 + 6(x-1) + 6(x-1)^2 + 6(x-1)^3$

(C) $6 + 6x + 3x^2 + x^3$

(D) $6 + 6(x-1) + 3(x-1)^2 + (x-1)^3$

(E) $6 + 3(x-1) + 1(x-1)^2 + \frac{1}{4}(x-1)^3$

n	$\frac{d}{dx}$	plug in 1	$n!$	$(x-c)^n$
0	$h(x)$	6	1	1
1	$h'(x)$	6	$1!$	$(x-1)$
2	$h''(x)$	6	$2!$	$(x-1)^2$
3	$h'''(x)$	6	$3!$	$(x-1)^3$

$6 + 6(x-1) + \frac{6(x-1)^2}{2!} + \frac{6(x-1)^3}{3!}$

$6 + 6(x-1) + 3(x-1)^2 + (x-1)^3$

16. E
30. B
31. D
35. C
37. A
44. C
45. A
47. C
48. D

#37)

$fg' + gf'$

n	$\frac{d}{dx}$	plug in 1	$n!$	$(x-c)^n$
0	$x \ln x$	$1 \cdot \ln 1 = 0$	1	1
1	$x \cdot \frac{1}{x} + \ln x \cdot 1$ $1 + \ln x$	1	$1!$	$(x-1)$
2	$\frac{1}{x} = x^{-1}$	1	$2!$	$(x-1)^2$
3	$\frac{-1}{x^2} = -1x^{-2}$	-1	$3!$	$(x-1)^3$
4	$2x^{-3} = \frac{2}{x^3}$	2	$4!$	$(x-1)^4$
5	$-6x^{-4} = \frac{-6}{x^4}$	-6	$5!$	$(x-1)^5$

$$0 + (x-1) + \frac{(x-1)^2}{2!} - \frac{(x-1)^3}{3!} + \frac{2(x-1)^4}{4!} - \frac{6(x-1)^5}{5!}$$

$$\frac{-6}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \boxed{\frac{-1}{20}}$$