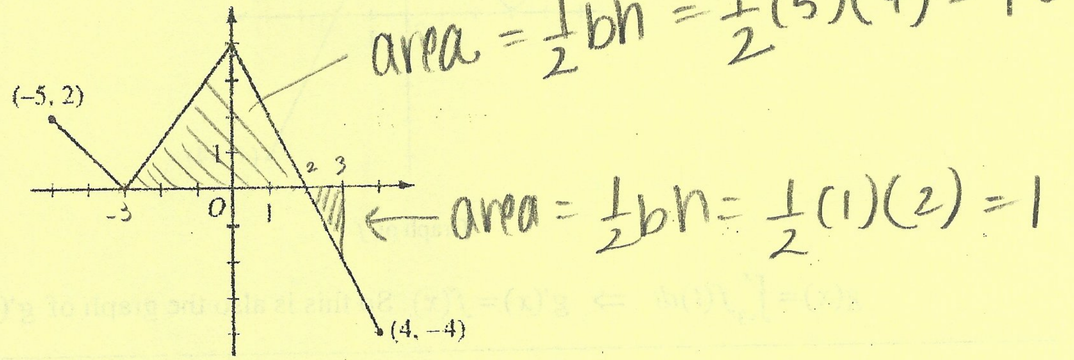
2014 AP® CALCULUS BC FREE-RESPONSE QUESTIONS

CALCULUS BC
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.



Graph of f

. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(3)$.

(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

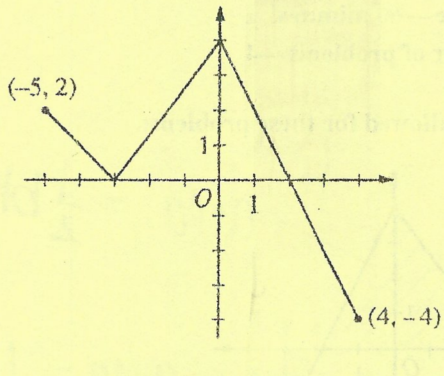
(c) The function h is defined by $h(x) = \dfrac{g(x)}{5x}$. Find $h'(3)$.

(d) The function p is defined by $p(x) = f\left(x^2 - x\right)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

2014 RELEASED FREE RESPONSE SOLUTIONS – MR. CALCULUS

2014 AB/BC #3

(no calculator)



Graph of f

$g(x)=\int_{-3}^{x} f(t)\,dt \Rightarrow g'(x)=f(x)$ So this is also the graph of $g'(x)$.

(a) $g(3)=\int_{-3}^{3} f(t)\,dt=\int_{-3}^{2} f(t)\,dt+\int_{2}^{3} f(t)\,dt=\boxed{\frac{1}{2}(5)(4)-\frac{1}{2}(1)(2)}=9$

(The area of the 2*nd* triangle is subtracted since it is below the *x*-axis.)

(b)

g is increasing when g' or $f>0 \Rightarrow -5<x<2$

g is concave down when g'' or $f'<0 \Rightarrow g'$ or f is decreasing $\Rightarrow -5<x<-3$ and $0<x<4$

Both occur when $\boxed{-5<x<-3 \text{ and } 0<x<2}$

(c)

$h(x)=\dfrac{g(x)}{5x} \Rightarrow h'(x)=\dfrac{5xg'(x)-5g(x)}{25x^2}$ using the quotient rule

$h'(3)=\dfrac{5(3)g'(3)-5g(3)}{25(9)}=\dfrac{15f(3)-5(9)}{25(9)}=\boxed{\dfrac{15(-2)-5(9)}{25(9)}}=-\dfrac{1}{3}$

(d)

$p(x)=f\left(x^2-x\right) \Rightarrow p'(x)=(2x-1)f'\left(x^2-x\right)$

The slope of the tangent line to p at $x=-1$ is

$p'(-1)=\left(2(-1)-1\right)f'\left((-1)^2-(-1)\right)=(-3)f'(2)=\boxed{(-3)\left(\dfrac{-4-4}{4-0}\right)}=6$

a) $g(3)$ use $g(x) = \int_{-3}^{x} f(t)dt$

$g(3) = \int_{-3}^{3} f(t)$ area under the curve f from -3 to 3

refer to graph

$= 10 + + -1 = \boxed{9}$

b) increasing means the slope is positive
$g'(x) > 0$

concave down means $g''(x) < 0$

if $g(x) = \int_{-3}^{x} f(t)dt$

$g'(x) = \frac{d}{dx} \int_{-3}^{x} f(t)dt$ Fundamental Thm of Calculus

$g'(x) = f(x)$ Thus $g''(x) = f'(x)$

Therefore, we need
$g'(x) = f(x) > 0$ and $g''(x) = f'(x) < 0$

$-5 < x < -3$ and
$-3 < x < 2$

$-5 < x < -3$ and
$0 < x < 4$

Together, we have
$-5 < x < -3$ and $0 < x < 2$

(c) $h(x) = \dfrac{g(x)}{5x}$ Find $h'(3)$

Quotent rule $\dfrac{gf' - fg'}{g^2}$

$$h'(x) = \frac{5x(g'(x)) - g(x) \cdot 5}{(5x)^2}$$

plug in 3

$$= \frac{5x \cdot g'(x) - 5g(x)}{25x^2}$$

$$= \frac{5(3) \cdot g'(3) - 5 \cdot g(3)}{25 \cdot 3^2}$$

$$= \frac{15 \cdot g'(3) - 5g(3)}{225}$$

$g(3) = 9$ from part a

$g'(x) = f(x)$

$g'(3) = f(3)$ → look on graph

$(3, -2)$

$g'(3) = -2$

$$= \frac{15 \cdot (-2) - 5 \cdot 9}{225}$$

$$= \frac{-30 - 45}{225} = \frac{-75}{225} = \boxed{-\frac{1}{3}}$$

d) $p(x) = f(x^2 - x)$

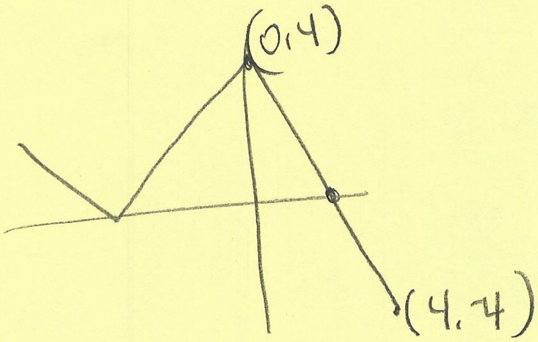
$p'(x) = f'(x^2 - x)\cdot(2x-1)$

$p'(-1) = f'((-1)^2 + 1)\cdot(2(-1)-1)$

$= f'(2)\cdot(-3) \quad = \quad -2\cdot(-3) = \boxed{6}$

$=$ ↑ slope at $x = 2$

pick endpoints of straight line



$m = \frac{-4-4}{4-0} = \frac{-8}{4} = -2$