

## Differentiation - Natural Logs and Exponentials

Differentiate each function with respect to  $x$ .

1)  $y = \ln x^3$

$$\frac{1}{x^3} \cdot 3x^2 = \frac{3x^2}{x^3} = \boxed{\frac{3}{x}}$$

2)  $y = e^{2x^3}$

$$e^{2x^3} \cdot 6x^2 = \boxed{6x^2 e^{2x^3}}$$

3)  $y = \ln(\ln 2x^4)$

$$\frac{1}{\ln 2x^4} \cdot \frac{1}{2x^4} \cdot 8x^3 = \frac{1}{\ln 2x^4} \cdot \frac{4}{x}$$

$$= \boxed{\frac{4}{x \ln 2x^4}}$$

4)  $y = \ln(\ln 3x^3)$

$$\frac{1}{\ln 3x^3} \cdot \frac{1}{3x^3} \cdot 9x^2 = \frac{1}{\ln 3x^3} \cdot \frac{3}{x}$$

$$= \boxed{\frac{3}{x \ln 3x^3}}$$

5)  $y = \cos(\ln 4x^3)$

$$-\sin(\ln 4x^3) \cdot \frac{1}{4x^3} \cdot 12x^2$$

$$= \boxed{\frac{-3 \sin(\ln 4x^3)}{x}}$$

6)  $y = e^{e^{3x^2}}$

$$e^{e^{3x^2}} \cdot e^{3x^2} \cdot 6x$$

$$= \boxed{6x e^{e^{3x^2} + 3x^2}}$$

7)  $y = e^{(4x^3+5)^2}$

$$e^{(4x^3+5)^2} \cdot 2(4x^3+5) \cdot 12x^2$$

$$= \boxed{24x^2 e^{(4x^3+5)^2} (4x^3+5)}$$

8)  $y = \ln 4x^2 \cdot (-x^3 - 4)$   $fg + gf'$

$$\ln(4x^2) \cdot -3x^2 + (-x^3 - 4) \cdot \frac{1}{4x^2} \cdot 8x$$

$$-3x^2 \ln(4x^2) + (-x^3 - 4) \cdot \frac{2}{x}$$

$$= \frac{-3x^3 \ln(4x^2) - 2x^3 - 8}{x}$$

$$= \boxed{\frac{-3x^3 \ln 4x^2 - 2x^3 - 8}{x}}$$

9)  $y = \ln\left(-\frac{4x^4}{x^3-3}\right)^5 = 5 \ln\left(-\frac{4x^4}{x^3-3}\right)$

$$5 \left( \frac{1}{-\frac{4x^4}{x^3-3}} \right)$$

10)  $y = \frac{e^{5x^4}}{e^{4x^2+3}}$

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$$= -\frac{3 \sin \ln 4x^3}{x}$$

6)  $y = e^{e^{3x^2}}$

$$\frac{dy}{dx} = e^{e^{3x^2}} e^{3x^2} \cdot 6x$$

$$= 6xe^{e^{3x^2} + 3x^2}$$

7)  $y = e^{(4x^3 + 5)^2}$

$$\frac{dy}{dx} = e^{(4x^3 + 5)^2} \cdot 2(4x^3 + 5) \cdot 12x^2$$

$$= 24x^2 e^{(4x^3 + 5)^2} (4x^3 + 5)$$

8)  $y = \ln 4x^2 \cdot (-x^3 - 4)$

$$\frac{dy}{dx} = \ln 4x^2 \cdot -3x^2 + (-x^3 - 4) \cdot \frac{1}{4x^2} \cdot 8x$$

$$= \frac{-3x^3 \ln 4x^2 - 2x^3 - 8}{x}$$

9)  $y = \ln \left( -\frac{4x^4}{x^3 - 3} \right)^5$

$$\frac{dy}{dx} = 5 \left( \frac{1}{-4x^4} \cdot -16x^3 - \frac{1}{x^3 - 3} \cdot 3x^2 \right)$$

$$= \frac{5(x^3 - 12)}{x(x^3 - 3)} \text{ (Rules of logarithms used)}$$

10)  $y = \frac{e^{5x^4}}{e^{4x^2 + 3}}$

$$\frac{dy}{dx} = e^{5x^4 - (4x^2 + 3)} (20x^3 - 8x)$$

$$= 4xe^{5x^4 - 4x^2 - 3} (5x^2 - 2) \text{ (Rules of exponents used)}$$

$$\#9) 5 \left( \frac{-\frac{1}{4x^4}}{x^3-3} \right) \cdot \left( \right)$$

$$\frac{gf' - fg'}{g^2} = 5 \left( \frac{\cancel{x^3-3}}{-4x^4} \right) \cdot \left[ \frac{(x^3-3)(-16x^3) - (-4x^4)(3x^2)}{(x^3-3)^2} \right]$$

$$= 5 \frac{1}{-4x^4} \cdot \left[ \frac{-16x^6 + 48x^3 + 12x^6}{(x^3-3)} \right]$$

$$= 5 \frac{1}{-4x^4} \left[ \frac{-4x^6 + 48x^3}{x^3-3} \right]$$

$$5 \frac{1}{\cancel{-4x^4} \cdot x} \left( \frac{\cancel{-4x^3}(x^3-12)}{x^3-3} \right)$$

$$\boxed{\frac{5(x^3-12)}{x(x^3-3)}}$$

#10)

$$\frac{e^{5x^4}}{e^{4x^2+3}}$$

$$\frac{gf' - fg'}{g^2}$$

$$e^{4x^2+3} \cdot e^{5x^4} \cdot 20x^3 - e^{5x^4} \cdot e^{4x^2+3} \cdot 8x$$

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$$(e^{4x^2+3})^2$$

$$20x^3 e^{4x^2+3+5x^4} - 8x e^{4x^2+3+5x^4}$$

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$$(e^{4x^2+3})^2$$

$$e^{4x^2+3} (20x^3 e^{5x^4} - 8x e^{5x^4})$$

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$$(e^{4x^2+3})^2$$

$$\frac{e^{5x^4} (20x^3 - 8x)}{e^{4x^2+3}}$$

$$e^{5x^4} e^{-(4x^2+3)} = e^{5x^4 - 4x^2 - 3} (20x^3 - 8x)$$

$$\boxed{4x e^{5x^4 - 4x^2 - 3} (5x^2 - 2)}$$