Answer key (1)

Calculus BC—Exam 1

Section I, Part A

Time: 55 minutes
Number of questions: 28

NO CALCULATOR MAY BE USED IN THIS PART OF THE EXAMINATION.

<u>Directions:</u> Solve each of the following problems. After examining the form of the choices, decide which is the best of the choices given.

<u>In this test:</u> Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

D 1. $\int_0^1 \sqrt{x}(x^2 + 1)\,dx =$

$x^{1/2}(x^2+1) = x^{5/2} + x^{1/2}$

(A) $\frac{4}{3}$ (B) $\frac{9}{7}$ (C) $\frac{16}{15}$

(D) $\frac{20}{21}$ (E) $\frac{4}{21}$

$\frac{2x^{7/2}}{7} + \frac{2x^{3/2}}{3}\Big|_0^1$

$\frac{2\cdot 3}{7\cdot 3} + \frac{2\cdot 7}{3\cdot 7}$

$\frac{6}{21} + \frac{14}{21} = \frac{20}{21}$

A 2. If $x = e^{4t}$ and $y = \sin 6t$, then $\frac{dy}{dx} =$

$dy \quad \frac{\cos(6t)\cdot 6}{e^{4t}\cdot 4}$

(A) $\frac{3e^{-4t}\cos 6t}{2}$. (B) $-\frac{3\cos 6t}{2e^{4t}}$. (C) $\frac{3e^{-4t}\cos t}{2}$.

(D) $e^{-4t}\cos 6t$. (E) $6\cos 6t$.

$2\cdot\frac{1}{4} - 1$

$(\frac{1}{2} - 1)$ $\quad \frac{3\cos(6t)}{2e^{4t}}$

C 3. The function f defined by $f(x) = x^4 - x^2$ has a relative minimum at $x =$

$2x(2x^2-1)$

(A) $\sqrt{2}$. (B) 1. (C) $\frac{\sqrt{2}}{2}$.

(D) $\frac{1}{2}$. (E) 0.

$4x^3 - 2x$

$-1 \quad 0 \quad \frac{1}{2} \quad \frac{\sqrt{2}}{2} \quad 5$

$- \quad - \quad \uparrow \quad +$

min

$x=0 \quad 2x^2-1=0 \; {+1}\;{+1} \quad 2x(2x^2-1)=0$

$\frac{2x^2}{2} = \frac{1}{2} \quad x = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$

$fg' + gf'$

$x^2 \cdot \cancel{e}^{\ln x^3} \cdot \frac{1}{x^3} \cdot 3x^2 + \cancel{e}^{\ln x^3} \cdot 2x$

4. $\frac{d}{dx} x^2 e^{\ln x^3} =$

$x^2 \cdot x^3 = x^5 = 5x^4$

(A) $6x^3$ (B) $5x^4$ (C) $2x + 3x^2$

(D) $2x^4 + x^5$ (E) $6x^4$

$x^2 \cdot \cancel{x^3} \cdot \frac{1}{\cancel{x^3}} \cdot 3x^2 + x^3 \cdot 2x$

$3x^4 + 2x^4$

$\boxed{5x^4}$

5. If $g(x) = \frac{1}{4}e^{2x-6} + (x-2)^{5/2}$, then $g'(3) =$

$\frac{1}{4}e^{2x-6} \cdot 2 + \frac{5}{2}(x-2)^{3/2}$

(A) 3. (B) $\frac{5}{2}$. (C) $\frac{11}{4}$.

(D) $\frac{1}{4}$. (E) 0.

$\frac{1}{4}e^0 \cdot 2 + \frac{5}{2}(1)^{3/2}$

$\frac{1}{2} + \frac{5}{2} = \frac{6}{2} = 3$

$fg' + gf'$

$2[x \cdot 2y \cdot \frac{dy}{dx} + y^2 \cdot 1]$

$+ 3\frac{1}{y}\frac{dy}{dx} = 2x - 9y^2\frac{dy}{dx}$

6. Find the slope of the line normal to the curve $y = -\sqrt{x+4}$ at the point where $x = 0$.

(A) -4 (B) $-\frac{1}{4}$ (C) $-\frac{1}{8}$

(D) $\frac{1}{4}$ (E) 4

$\frac{dy}{dx} = -\frac{1}{2}(x+4)^{-1/2} \cdot 1$

$4xy \cdot \frac{dy}{dx} + 2y^2 + \frac{3}{y}\frac{dy}{dx} = 2x - 9y^2\frac{dy}{dx}$

7. Compute $\frac{dy}{dx}$ for the relation $2xy^2 + 3\ln y = x^2 - 3y^3$ at the point (3, 1).

$12\frac{dy}{dx} + 2 + 3\frac{dy}{dx} = 6 - 9\frac{dy}{dx}$

$15\frac{dy}{dx} + 2 = 6 - 9\frac{dy}{dx}$
$+9 \quad -2 \quad -2 \quad +9$

$\frac{24}{24}\frac{dy}{dx} = \frac{4}{24} \quad \frac{1}{6}$

(A) $\frac{5}{2}$ (B) $\frac{3}{4}$ (C) $\frac{5}{12}$

(D) $\frac{1}{8}$ (E) $\frac{1}{6}$

$-\frac{1}{2} \frac{1}{\sqrt{0+4}}$

$-\frac{1}{4}$

4
perpendicular

$u = 1 + x^3$

$\frac{du}{3} = \frac{3x^2}{3}$

$\frac{1}{3}du = x^2 dx$

D

8. $\int_1^\infty \frac{x^2}{(1+x^3)^2}\,dx$ is

$\frac{1}{3}\int_1^\infty \frac{1}{u^2}\,du \quad u^{-2+1}$

(A) $-\frac{1}{6}$. (B) $-\frac{1}{24}$. (C) $\frac{1}{24}$.

(D) $\frac{1}{6}$. (E) divergent.

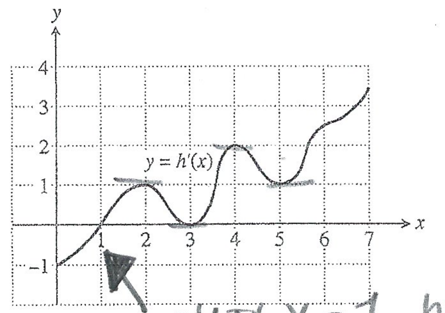
$\frac{1}{3}\frac{u^{-1}}{-1}\Big|_\infty^0$

$\frac{1}{3(1+1)}$

3·2

$\frac{-1}{3u}\Big|_0^\infty \quad \frac{-1}{\cancel{3(\infty)}} + \frac{1}{3(1+1^3)} \quad \frac{1}{6}$

The function $h(x)$ is continuous and differentiable on the domain $[0, 7]$. The graph of $h'(x)$ is shown. Use the graph for Questions 9, 10, and 11.



after $x=1$ the slope is positive

9. At what value of x does $h(x)$ have its absolute minimum?

so $x=1$ is the lowest point

(A) 0 (B) 1 (C) 3

(D) 5 (E) 7

10. The point $(5, 2)$ is on the graph of $y = h(x)$. An equation of the line tangent to $h(x)$ at $(5, 2)$ is

slope at $x=5$ is 1

(A) $y - 2 = x - 5$. (B) $y = x - 2$.

(C) $y - 2 = 2(x - 5)$. (D) $x = 5$.

(E) $y = 2$.

$y-2=1(x-5)$

$y-2=x-5$

$+2 \quad +2$

$y=x-3$

11. How many inflection points does h have on the interval $(0, 7)$?

(A) 3 (B) 4 (C) 5

(D) 6 (E) 7

where does $h''(x)=0$

so where does the slope of $h'(x)=0$ how many

12. The sum of the infinite geometric series $\frac{8}{25} - \frac{24}{125} + \frac{72}{625} - \frac{216}{3125} + \cdots$ is

must be a geometric series

$\frac{a_1}{1-r}$

(A) 0.2. (B) 0.6. (C) 0.8.

(D) 1.0. (E) 1.2.

$\left(\frac{-3}{5}\right)$

$\frac{\frac{8}{25}}{\frac{8}{5}}$

$\frac{\frac{8}{25}}{1+\frac{3}{5}}$

$\frac{8}{25} \cdot \frac{5}{8} = \frac{1}{5} = .\frac{2}{10} = .2$

13. A particle moves along the x-axis so that its acceleration at any time t is $a(t) = 2t - 3$. If the initial velocity of the particle is -4, at what time t in the time interval $0 \leq t \leq 5$ is the particle farthest left?

(A) 0 (B) $\frac{3}{2}$ (C) 3

(D) 4 (E) 5

$\int 2t-3$

$\frac{2t^2}{2} - 3t + C$

$v(t) = t^2 - 3t + C$

at $t=0$, $v=-4$

$-4 = 0^2 - 3(0) + C$

$C = -4$

$v(t) = t^2 - 3t - 4 = 0$

see when we stop and change direction

$(t-4)(t+1) = 0$

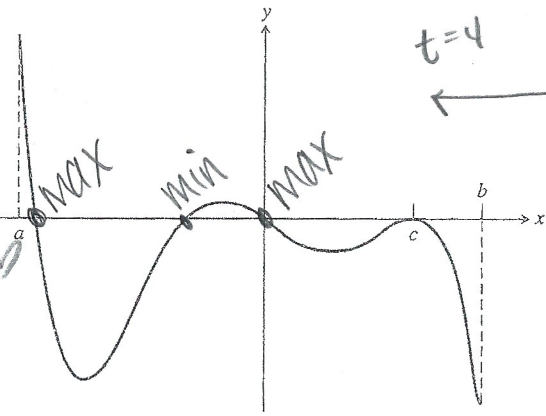
$t=4$, $t=\cancel{-1}$

$t=0$ $t=4$ $t=5$

moving left moving right

$t=4$ $t=0$

14. The graph of $f'(x)$ is shown. It is tangent to the x-axis at point c. Which of the following describes all relative extrema of $f(x)$ on the open interval (a, b)?



(A) One relative maximum and one relative minimum

(B) One relative maximum and two relative minima

(C) Three relative maxima and two relative minima

~~(D) Two relative maxima and two relative minima~~

(E) Two relative maxima and one relative minimum

15. The length of the path described by the parametric equations $x = 2\sin t$ and $y = 3\cos t$ for $0 \leq t \leq \pi/2$ is given by

(A) $\int_0^{\pi/2} \sqrt{4 + 5\sin^2 t}\, dt.$

(B) $\int_0^{\pi/2} \sqrt{4\cos^2 t - 9\sin^2 t}\, dt.$

(C) $\int_0^{\pi/2} \sqrt{4\sin^2 t + 9\cos^2 t}\, dt.$

(D) $\int_0^{\pi/2} \sqrt{1 + \frac{9\sin^2 t}{4\cos^2 t}}\, dt.$

(E) none of the above.

$\cancel{s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx}$

$\cancel{\int_0^{\pi/2} \sqrt{1 + \left(\frac{-3\sin t}{2\cos t}\right)^2}}$

$\cancel{\int_0^{\pi/2} \sqrt{1 + \frac{9\sin^2 t}{4\cos^2 t}}\, d}$

distance formula

$\int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$(2\cos t)^2 + (-3\sin t)^2$

$\sqrt{4\cos^2 t + 9\sin^2 t}\, dt$

16. $\lim_{x \to 3} \frac{e^{x^2} - e^9}{x - 3} =$

$\frac{e^9 - e^9}{3-3} = \frac{0}{0}$ L'Hopitals

$e^9 \cdot 6$

$6e^9$

$\frac{e^{x^2} \cdot 2x - 0}{1}$

(A) 0
(B) $\frac{e^9}{3}$
(C) $3e^9$
(D) $6e^9$
(E) ∞

17. Let f be the function defined by $f(x) = \ln(x + 4)$. The third-degree Taylor polynomial for f centered about $x = -3$ is

(A) $x - 3 - \frac{(x-3)^2}{2} - \frac{(x-3)^3}{3}$.

(B) $x - 3 - \frac{(x-3)^2}{2} + \frac{(x-3)^3}{3}$.

(C) $-3 + x - \frac{(x+3)^2}{2} + \frac{(x+3)^3}{3}$.

(D) $3 + x + \frac{(x+3)^2}{2} + \frac{(x+3)^3}{3}$.

(E) $3 + x - \frac{(x+3)^2}{2} + \frac{(x+3)^3}{3}$.

n	$f^n(x)$	$f^n(c)$ $x=-3$	$(x-c)^n$	$n!$
0	$\ln(x+4)$	0	$(x+3)^0=1$	1
1	$\frac{1}{x+4}$ $(x+4)^{-1}$	1	$(x+3)^1$	$1!$
2	$\frac{-1}{(x+4)^2}$ $-1(x+4)^{-2}$	-1	$(x+3)^2$	$2!$
3	$\frac{2}{(x+4)^3}$	2	$(x+3)^3$	$3!$

$\frac{x+3}{1!} - \frac{(x+3)^2}{2!} + \frac{2(x+3)^3}{3!}$

18. For what values of t does the curve defined by the parametric equations $x = \frac{4}{3}t^3 - t^2$ and $y = t^5 + t^2 - 7t$ have a vertical tangent?

(A) 0 only
(B) 0 and $\frac{1}{2}$
(C) $\frac{1}{2}$ only
(D) 1 only
(E) $0, \frac{1}{2}$, and 1

when does $dx = 0$

$4t^2 - 2t = 0$

$2t(2t-1) = 0$

$t = 0 \quad t = \frac{1}{2}$

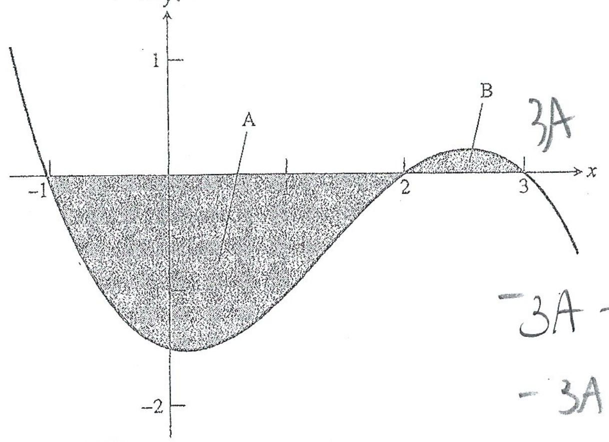
19. The graph of $y = f(x)$ is shown. Let A and B be positive numbers that represent the area of each shaded region. Evaluate $\int_3^{-1} f(x)\,dx + 3\int_{-1}^{2} f(x)\,dx$ in terms of A and B.

$+3A$

$-\int_{-1}^{3} f(x)$

$-\left[\int_{-1}^{2} + \int_{2}^{3}\right.$

$A + B$



$3A - A - B$

A is negative area

$-3A - -A - B$

$-3A + A - B$

$-2A - B$

(A) $-2A - B$ (B) $2A + B$ (C) $3A - B$

(D) $3A + B$ (E) $-3A - B$

Interval of convergence

20. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n \cdot 4^n}$ converges?

(A) $-\frac{3}{2} \le x \le \frac{5}{2}$ (B) $-\frac{3}{2} \le x < \frac{5}{2}$

(C) $-\frac{3}{2} < x \le \frac{5}{2}$ (D) $-3 \le x < 5$

(E) $-3 < x \le 5$

$\lim_{n\to\infty} \left| \frac{(2x-1)^{n+1}}{(n+1)\cdot 4^{n+1}} \cdot \frac{n \cdot 4^n}{(2x-1)^n} \right|$

$\lim_{n\to\infty} \left| \frac{(2x-1)^n \cdot (2x-1)}{(n+1)\cdot 4^n \cdot 4^1} \cdot \frac{n \cdot 4^n}{(2x-1)^n} \right|$

$\lim_{n\to\infty} \left| \frac{1n}{4n+4} \cdot 2x-1 \right|$

$4 \cdot \frac{1}{4}|2x-1| < 1 \cdot 4$

$-4 < 2x-1 < 4$
$+1 \quad +1 \quad +1$

$-\frac{3}{2} < \frac{2x}{2} < \frac{5}{2}$

$-\frac{3}{2} < x < \frac{5}{2}$

cos = 0 when $\pi/2$, $\frac{3\pi}{2}$

$2\theta = \frac{\pi}{4}$

when $r = 0$

$\frac{0}{3} = \frac{3\cos 2\theta}{3}$

$0 = \cos 2\theta$

21. The expression representing the area inside one leaf of the polar rose $r = 3\cos 2\theta$ is given by

(A) $\int_0^{\pi/4} \sqrt{1 + 9\sin^2 2\theta}\, d\theta$ (B) $\int_0^{\pi/2} \sqrt{1 + 9\sin^2 2\theta}\, d\theta$

(C) $\int_0^{\pi/2} 9\cos^2 2\theta\, d\theta$ (D) $\frac{1}{2}\int_0^{\pi/4} 9\cos^2 2\theta\, d\theta$

(E) $\int_0^{\pi/4} 9\cos^2 2\theta\, d\theta$

$A = \int \frac{1}{2} r^2\, d\theta$

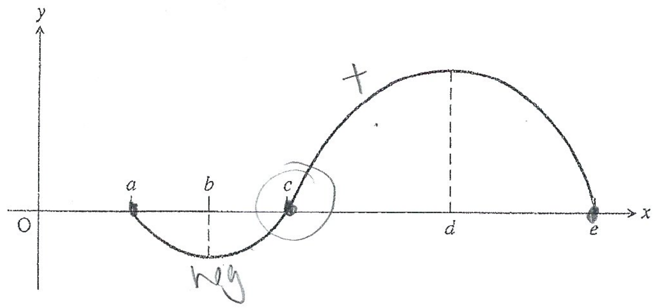
½ leaf

$2 \cdot \frac{1}{2}\int_0^{\pi/4} (3\cos 2\theta)^2\, d\theta$

$x = \frac{5}{2}$ $\frac{(5-1)^n}{n\cdot 4^n} = \frac{4^n}{n \cdot 4^n} = \frac{1}{n}$ diverges p-series $p = 1$

2θ $\int_0^{\pi/4} \frac{1}{2}(3\cos(2\theta))^2 - \int_0^{\pi/4} \frac{1}{2} 9\cos^2(2\theta)$

22.



The graph of f is shown. If $h(x) = \int_a^x f(t)\,dt$, for what value of x does $h(x)$ have its minimum?

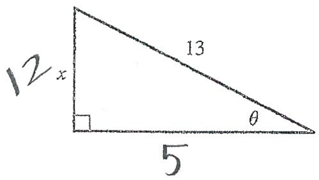
$h'(x) = f(x) = 0$

(A) a (B) b (C) c

(D) d (E) e

23. In the right triangle shown, θ is increasing at a constant rate of 2 radians per minute. In units per minute, at what rate is x increasing when $x = 12$?

$\frac{d\theta}{dt} = 2$



$\sin\theta = \frac{x}{13}$

$13\sin\theta = x$

(A) 2 (B) 4 (C) 5

(D) 10 (E) 24

$13\cos\theta \cdot \frac{d\theta}{dt} = 1\,\frac{dx}{dt}$

$10 = \cancel{13} \cdot \frac{5}{\cancel{13}} \cdot 2 = \frac{dy}{dt}$

24. The Taylor series for $\cos x$ about $x = 0$ is $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$. If h is a function such that $h'(x) = \cos x^3$, then the coefficient of x^7 in the Taylor series for $h(x)$ about $x = 0$ is

$-24 = \frac{dx}{dt}$

(A) $-\frac{1}{14}$. (B) $-\frac{1}{7!}$. (C) 0.

(D) $\frac{1}{7!}$. (E) $\frac{1}{14}$.

$\cos x^3 = 1 - \frac{(x^3)^2}{2!} + \frac{(x^3)^4}{4!}$

$h'(x) = \int 1 - \int \frac{x^6}{2!} + \frac{x^{12}}{4!}$

to find $h(x)$ integrate

$-\frac{x^7}{2! \cdot 7}$ $\quad -\frac{1}{14}$

pg#298

$\lim_{n\to\infty} \sum_{K=1}^{n} \frac{1}{2} r_K^2 \Delta\theta_K$

$= \int_a^b \frac{1}{2} r^2 \, d\theta$

25. The closed interval $[w, v]$ is partitioned into k equal subintervals, each with width Δx, by the numbers $x_0, x_1, \ldots, x_k$ with $w = x_0 < x_1 < x_2 < \cdots < x_{k-1} < x_k = v$. The $\lim_{k\to\infty} \sum_{j=1}^{k} \frac{1}{\sqrt{x_j}} \Delta x$ equals

(A) $\sqrt{v} - \sqrt{w}$. (B) $2(\sqrt{v} - \sqrt{w})$.

(C) $\frac{1}{\sqrt{v}} - \frac{1}{\sqrt{w}}$. (D) $2\left(\frac{1}{\sqrt{v}} - \frac{1}{\sqrt{w}}\right)$.

(E) $\frac{2}{3}\left(v^{3/2} - w^{3/2}\right)$.

$\int_W^V \frac{1}{\sqrt{X}} dX = 2\sqrt{X}\Big|_W^V$

$2\sqrt{V} - 2\sqrt{W}$

$\frac{A}{(X-3)}^{X+2} + \frac{B}{X+2}^{X-3}$

26. $\int \frac{6x - 8}{(x - 3)(x + 2)} dx =$

(A) $2 \ln|x - 3| + 4 \ln|x + 2| + C$

(B) $2 \ln|x - 3| + 2 \ln|x + 2| + C$

(C) $2 \ln|x + 3| + 4 \ln|x - 2| + C$

(D) $6x \ln|x - 3| + 8 \ln|x + 2| + C$

(E) $4 \ln|x - 3| + 2 \ln|x + 2| + C$

$\int \frac{2}{X-3} + \frac{4}{X+2} =$

$2\ln|X-3| + 4\ln|X+2| + C$

$AX + 2A + BX - 3B$

$(A+B)X + 2A - 3B = 6X - 8$

$-2(A+B=6)$

$+2A-3B=-8$

$-2A+2B=-12$

$+2A-3B=-8$

$-5B=-20$

$\boxed{B=4}$

$A+4=6$

$A=2$

27. $\int 2x \cos x \, dx =$ integration by parts

(A) $x^2 \sin x + C$ (B) $x^2 \cos \frac{x^2}{2} + C$

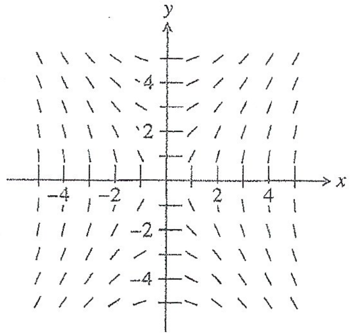
(C) $2 \sin x - 2x \cos x + C$ (D) $-2x \sin x - 2 \cos x + C$

(E) $2x \sin x + 2 \cos x + C$

D	I
$2X$	$\cos X$
+	
2	$\sin X$
−	
0	$-\cos X$

$2X\sin X + 2\cos X + C$

28. Which of the following equations has the slope field shown?



(0,#) = 0

(A) $\cancel{\dfrac{dy}{dx} = 2x}$ = −

(B) $\cancel{\dfrac{dy}{dx} = 2y}$

(C) $\dfrac{dy}{dx} = \dfrac{2x}{y}$ +

(D) $\cancel{\dfrac{dy}{dx} = xy}$ +

(E) $\cancel{\dfrac{dy}{dx} = \dfrac{2y}{x}}$

(-2, -2)

Calculus BC–Exam 1

Section I

Part A – No calculator

Problem	Answer
1.	(D)
2.	(A)
3.	(C)
4.	(B)
5.	(A)
6.	(E)
7.	(E)
8.	(D)
9.	(B)
10.	(A)
11.	(B)
12.	(A)
13.	(D)
14.	(E)
15.	C
16.	(D)
17.	(E)
18.	(B)
19.	(A)
20.	(B)
21.	(E)
22.	(C)
23.	(D)
24.	(A)
25.	(B)
26.	(A)
27.	E
28.	(C)

Part B – Calculator allowed

Problem	Answer
29.	(B)
30.	(D
31.	(E)
32.	(D)
33.	(B)
34.	(C)
35.	(B)
36.	(A)
37.	(D)
38.	(D)
39.	(A)
40.	(C)
41.	(D)
42.	(B)
43.	(E)
44.	(E)
45.	(D)