BC#1

39

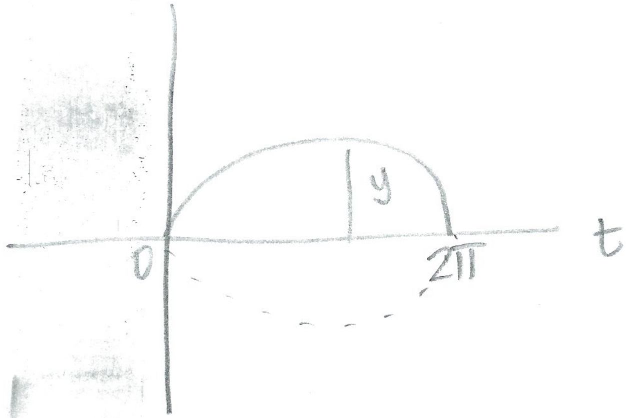
39. Find the volume of the solid formed when one arch of the cycloid defined parametrically by $x = \theta - \sin\theta$, $y = 1 - \cos\theta$ is rotated around the x-axis.

(A) 15.708 (B) 17.306 (C) 19.739 (D) 29.609 (E) 49.348

change MODE : Func. to PAR

graph $x_1 = T - \sin T$

$y_1 = 1 - \cos T$



Find area of rotation is a circle

πr^2 where $r = y$

$$\int_0^{2\pi} \pi y^2 \, dx \qquad y = 1 - \cos t$$

→ Find derivative of x

$x = t - \sin t$

$dx = (1 - \cos t)\, dt$

$$\pi \int_0^{2\pi} (1 - \cos t)^2 (1 - \cos t)\, dt$$

$$\pi \int_0^{2\pi} (1 - \cos t)^3 \, dt$$

plug into calculator

$\boxed{49.348}$ E

BC #1

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40. Which definite integral represents the length of the first quadrant arc of the curve defined by $x(t) = e^t$, $y(t) = 1 - t^2$?

(A) $\int_{-1}^{1} \sqrt{1 + \frac{4t^2}{e^{2t}}}\,dt$ (B) $\int_{1/e}^{e} \sqrt{1 + \frac{4t^2}{e^{2t}}}\,dt$ (C) $\int_{-1}^{1} \sqrt{e^{2t} + 4t^2}\,dt$

(D) $\int_{0}^{1} \sqrt{e^{2t} + 4t^2}\,dt$ (E) $\int_{1/e}^{e} \sqrt{e^{2t} + 4t^2}\,dt$

For the first quadrant, we know that both x and y must be positive

* we know that $x = e^t$ is positive for all t
* we know that $y = 1 - t^2$ is only positive when $-1 < t < 1$

The formula for arc length is

$$\int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$x = e^t$ $y = 1 - t^2$

$\boxed{\frac{dx}{dt} = e^t}$ $\boxed{\frac{dy}{dt} = -2t}$ plug into formula.

when both x, y are positive

$$\int_{-1}^{1} \sqrt{(e^t)^2 + (-2t)^2}\,dt$$

$$\boxed{\int_{-1}^{1} \sqrt{e^{2t} + 4t^2}\,dt}$$ C