BC #1

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35. In a marathon, when the winner crosses the finish line many runners are still on the course, some quite far behind. If the density of runners x miles from the finish line is given by $R(x) = 20[1 - \cos(1 + 0.03x^2)]$ runners per mile, how many are within 8 miles of the finish line?

(A) 30 (B) 145 (C) 157 (D) 166 (E) 195

* looking for how many runners are 0 miles to 8 miles from the finish line

Thus integrate

$$\int_0^8 20[1-\cos(1+0.03x^2)]\,dx$$

plug into your calc

MATH → fnInt(20(1-cos(1+0.03x²)), x, 0, 8)

$= 166.396$

$= \boxed{166}$ D

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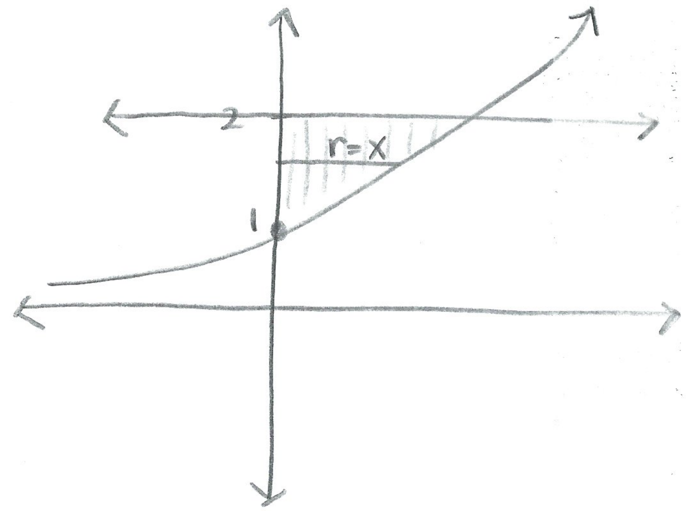
36. Find the volume of the solid generated when the region bounded by the y-axis, $y = e^x$, and $y = 2$ is rotated around the y-axis.

(A) 0.296 (B) 0.592 (C) 2.427 (D) 3.998 (E) 27.577

graph $y = e^x$ and $y = 2$



when rotated around the y-axis, we get a circle shape, thus we will use the formula πr^2

with $r = x$, we have πx^2

since we are integrating along the y-axis from 1 to 2, we need to write $y = e^x$ in terms of y

$\ln y = \ln e^x$ $\ln y = x$

now plug in

$$\int_1^2 \pi (\ln y)^2 \, dy = \boxed{.592} \quad B$$

plug in

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37. If $f(t) = \int_0^{t^2} \frac{1}{1+x^2}\,dx$, then $f'(t)$ equals

(A) $\frac{1}{1+t^2}$ (B) $\frac{2t}{1+t^2}$ (C) $\frac{1}{1+t^4}$ (D) $\frac{2t}{1+t^4}$ (E) $\tan^{-1} t^2$

$$\frac{d}{dt} f(t) = \frac{d}{dt}\int_0^{t^2} \frac{1}{1+x^2}\,dx$$

* take derivative of both sides

plug t^2 into x, times the derivative of t^2

$$f'(t) = \frac{1}{1+(t^2)^2} \cdot 2t$$

$$= \boxed{\frac{2t}{1+t^4}}$$

D

BC #1

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38. You wish to estimate e^x, over the interval $|x| < 2$, with an error less than 0.001. The Lagrange error term suggests that you use a Taylor polynomial at 0 with degree at least

(A) 6 (B) 9 (C) 10 (D) 11 (E) 12

we know that

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

The Lagrange remainder R, after n terms, for some c in the interval $0<c<x$ definition is

$$R = \frac{f^{(n+1)}(c)\cdot c^{n+1}}{(n+1)!} \quad \text{for } 0<c<x$$

Now lets plug in our information

$f(x) = e^x$, so $f(c) = e^c$, & since $|x| < 2$

$$R = \frac{e^c \cdot c^{n+1}}{(n+1)!} \qquad 0<c<2$$

so, if $0<c<2$, then R would be the greatest value when $c=2$, so plug in $c=2$ and n needs to satisfy

given error

$$\frac{e^2\cdot 2^{n+1}}{(n+1)!} < 0.001$$

now plug in different values for n.

~~A)~~ $n=6$ $\quad \frac{e^2\cdot 2^{6+1}}{(6+1)!} = .187$ not < 0.001

~~B)~~ $n=9$ $\quad \frac{e^2\cdot 2^{9+1}}{(9+1)!} = .0021$ not < 0.001

c) $n=10$ $\quad \frac{e^2\cdot 2^{10+1}}{(10+1)!} =$

0.000379

which is less than 0.001 ☺

$\boxed{C}$