SECTION II

Part A TIME: 30 MINUTES
2 PROBLEMS

A graphing calculator is required for some of these problems.
See instructions on page 4.

1. Let function f be continuous and decreasing, with values as shown in the table:

x	2.5	3.2	3.5	4.0	4.6	5.0
$f(x)$	7.6	5.7	4.2	3.8	2.2	1.6

(a) Use the trapezoid method to estimate the area between f and the x-axis on the interval $2.5 \le x \le 5.0$.

(b) Find the average rate of change of f on the interval $2.5 \le x \le 5.0$.

(c) Estimate the instantaneous rate of change of f at $x = 2.5$.

(d) If $g(x) = f^{-1}(x)$, estimate the slope of g at $x = 4$.

2. An object starts at point (1,3), and moves along the parabola $y = x^2 + 2$ for $0 \le t \le 2$, with the horizontal component of its velocity given by $\dfrac{dx}{dt} = \dfrac{4}{t^2+4}$.

(a) Find the object's position at $t = 2$.

(b) Find the object's speed at $t = 2$.

(c) Find the distance the object traveled during this interval.

STOP

END OF PART A, SECTION II

Part B TIME: 60 MINUTES

4 PROBLEMS

No calculator is allowed for any of these problems.
If you finish Part B before time has expired, you may return to work on Part A, but you may not use a calculator.

3. Given a function f such that $f(3) = 1$ and $f^{(n)}(3) = \dfrac{(-1)^n n!}{(2n+1)2^n}$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f around $x = 3$.

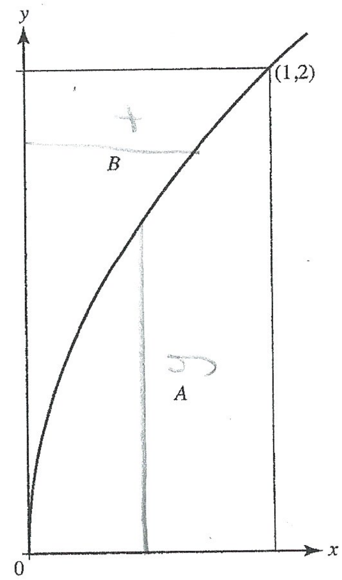
(b) Find the radius of convergence of the Taylor series.

(c) Show that the third-degree Taylor polynomial approximates $f(4)$ to within 0.01.

4. The curve $y = \sqrt{8\sin\left(\dfrac{\pi x}{6}\right)}$ divides a first quadrant rectangle into regions $\boldsymbol{A}$ and $\boldsymbol{B}$, as shown in the figure.

(a) Region $\boldsymbol{A}$ is the base of a solid. Cross sections of this solid perpendicular to the x-axis are rectangles. The height of each rectangle is 5 times the length of its base in region $\boldsymbol{A}$. Find the volume of this solid.

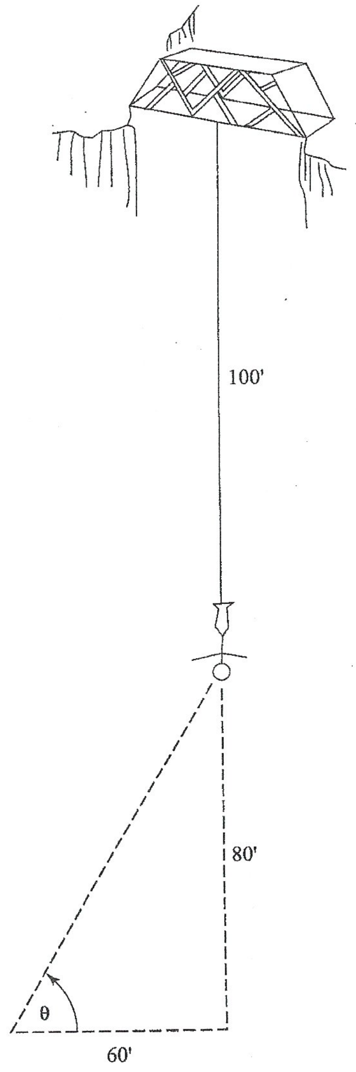
(b) The other region, $\boldsymbol{B}$, is rotated around the y-axis to form a different solid. Set up but do not evaluate an integral for the volume of this solid.

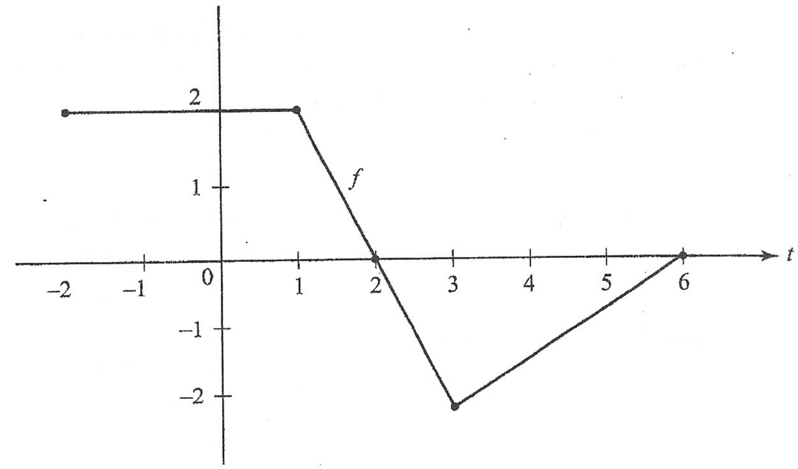


5. A bungee jumper has reached a point in her exciting plunge where the taut cord is 100 feet long with a 1/2-inch radius, and stretching. She is still 80 feet above the ground and is now falling at 40 feet per second. You are observing her jump from a spot on the ground 60 feet from the potential point of impact, as shown in the diagram above.

(a) Assuming the cord to be a cylinder with volume remaining constant as the cord stretches, at what rate is its radius changing when the radius is 1/2″?

(b) From your observation point, at what rate is the angle of elevation to the jumper changing when the radius is 1/2″?





6. The figure above shows the graph of f, whose domain is the closed interval $[-2,6]$. Let $F(x)=\int_1^x f(t)\,dt$.

(a) Find $F(-2)$ and $F(6)$.

(b) For what value(s) of x does $F(x)=0$?

(c) For what value(s) of x is F increasing?

(d) Find the maximum value and the minimum value of F.

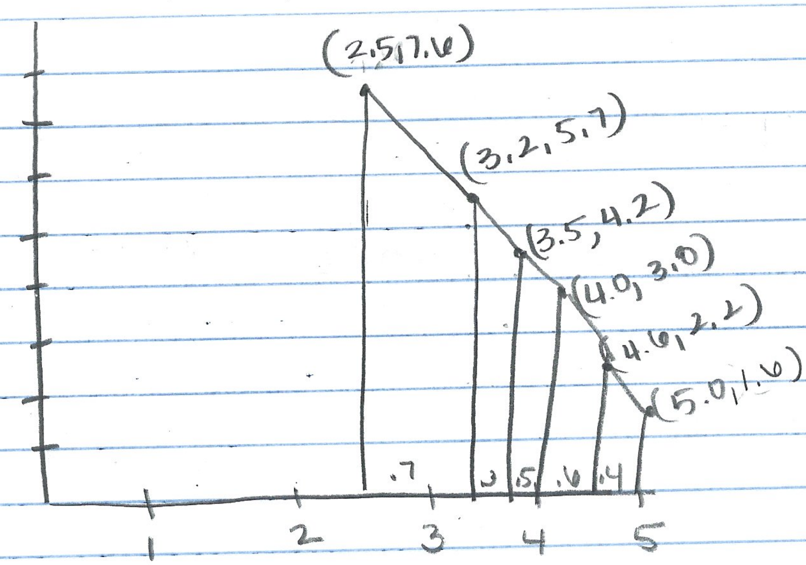
(e) At what value(s) of x does the graph of F have points of inflection? Justify your answer.

STOP

END OF TEST

FRQ BC Practice Exam # 2

1 a)



Area of a Trapezoid $= \frac{1}{2}h(b_1 + b_2)$

$$\frac{1}{2}(.7)(7.6+5.7) + \frac{1}{2}(.3)(5.7+4.2) + \frac{1}{2}(.5)(4.2+3.0)$$

$$+\frac{1}{2}(.6)(3.0+2.2) + \frac{1}{2}(.4)(2.2+1.6)$$

$$= \boxed{10.7}$$

b) slope (2.5, 7.6) and (5.0, 1.6)

$$m = \frac{7.6-1.6}{2.5-5} = \frac{6.0}{2.5} = \boxed{-2.4}$$

c) slope at 2.5, pick the closest point

(2.5, 7.6) and (3.2, 5.7)

$$\frac{7.6-5.7}{2.5-3.2} = \boxed{-2.714}$$

a) $g(x) = f^{-1}(x)$

switch the x and y coordinates

x	7.6	5.7	4.2	3.8	2.2	1.6
$f(x)$	2.5	3.2	3.5	4.0	4.6	5.0

pick and two pairs

Find the slope at 4

$$\frac{4.0 - 3.5}{3.8 - 3.5} = \boxed{-1.25}$$

② velocity $\frac{dx}{dt} = \frac{4}{t^2+4}$

$\frac{1}{t^2+4}$

$\frac{1}{4}\frac{1}{\left(\frac{t^2}{4}+1\right)}$

a) $\int \frac{dx}{4} = \int \frac{1}{t^2+4}\,dt$

$\frac{X}{4} = \int \frac{1}{4} \cdot \frac{1}{\left(\frac{t^2}{4}+1\right)}\,dt$

$\frac{1}{4}\ \frac{1}{\left(\frac{t}{2}\right)^2+1}$

$u = \frac{t}{2}$

$2du = \frac{1}{2}du$

$2du = dt$

$\frac{X}{4} = \frac{1}{4}\int \frac{1}{\left(\frac{t}{2}\right)^2+1}$

$\frac{X}{4} = 2 \cdot \frac{1}{4}\int \frac{1}{u^2+1}\,du$

$\frac{X}{4} = \frac{1}{2}\tan^{-1} u + C$

$4\ \frac{X}{4} = \frac{4 \cdot 1}{2}\tan^{-1}\frac{t}{2} + C$

$X = 2\tan^{-1}\left(\frac{t}{2}\right) + C$

we know at $t=0$ $\overset{x\ \ y}{(1, 3)}$

lets find c

$\tan^{-1}(0) = 0$

$1 = 2\tan^{-1}\left(\frac{0}{2}\right) + c$

$X = 2\tan^{-1}\left(\frac{t}{2}\right) + 1$

$1 = 2\tan^{-1}(0) + C$

$1 = 2(0) + C$

$C = 1$

$X = 2\tan^{-1}\left(\frac{t}{2}\right)+1$

Find x-coordinate at $t=2$

$X = 2\tan^{-1}\left(\frac{2}{2}\right)+1$

$= 2\tan^{-1}(1)+1$

$= 2\cdot\frac{\pi}{4}+1$

$X = \frac{\pi}{2}+1$

$y = x^2+2$

since $x = \frac{\pi}{2}+1$

$y = \left(\frac{\pi}{2}+1\right)^2+2$

$\boxed{\left(\frac{\pi}{2}+1,\ \left(\frac{\pi}{2}+1\right)^2+2\right)}$

b) Speed $= \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$

$\boxed{= \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\pi}{2}+1\right)^2}}$

$\frac{dx}{dt} = \frac{4}{t^2+4}$ $t=2$

$\frac{dx}{dt} = \frac{4}{2^2+4} = \frac{4}{8} = \frac{1}{2}$

$\frac{dx}{dt} = \frac{1}{2}$

c) Distance traveled

$t=0$ start, $t=2$ end

$1 < x < \frac{\pi}{2}+1$

$\int \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

$\int_1^{\frac{\pi}{2}+1} \sqrt{1+(2x)^2}\,dx$

$\int_1^{\frac{\pi}{2}+1} \sqrt{1+4x^2}\,dx$

on calc $= \boxed{5.839}$

Find $\frac{dy}{dx}$

$y = x^2+1$

$dy = 2x\,dx$

$\frac{dy}{dx} = 2x$

since $y = x^2+2$

$\frac{dy}{dt} = 2x\frac{dx}{dt}$

$\frac{dy}{dt} = 2\cdot\left(\frac{\pi}{2}+1\right)\left(\frac{1}{2}\right)$

$\frac{dy}{dt} = \frac{\pi}{2}+1$

(3) $f(3)=1 \qquad f^{(n)}(3) = \dfrac{(-1)^n n!}{(2n+1)\,2^n}$

a) $n=0 \qquad \dfrac{(-1)^0\, 0!}{(2(0)+1)\,2^0} = \dfrac{1}{1\cdot 1} = 1$

$n=1 \qquad \dfrac{(-1)^1\, 1!}{(2(1)+1)\,2^1} = \dfrac{-1}{3\cdot 2^1} = \dfrac{-1}{3\cdot 2^1} = \dfrac{-1}{6\cdot 1!}$

$n=2 \qquad \dfrac{(-1)^2\cdot 2!}{(2(2)+1)\,2^2} = \dfrac{2!}{5\cdot 2^2} = \dfrac{\cancel{2!}}{5\cdot 2^2\cdot \cancel{2!}} = \dfrac{1}{5\cdot 2^2} = \dfrac{1}{20}$

$n=3 \qquad \dfrac{(-1)^3\cdot 3!}{(2(3)+1)\,2^3} = \dfrac{-1\,\cancel{3!}}{7\cdot 2^3\cdot \cancel{3!}} = \dfrac{-1}{7\cdot 2^3} = -\dfrac{1}{56}$

$$1 - \frac{1}{6}(x-3) + \frac{1}{20}(x-3)^2 - \frac{1}{56}(x-3)^3 + \cdots + \frac{(-1)^n (x-3)^n}{(2n+1)\,2^n} + \cdots$$

b) $\frac{(-1)^n (x-3)^n}{(2n+1)\,2^n}$

ratio Test

$$\lim_{n\to\infty} \left| \frac{(x-3)^{n+1}}{(2(n+1)+1)2^{n+1}} \cdot \frac{(2n+1)2^n}{(x-3)^n} \right|$$

$$\lim_{n\to\infty} \left| \frac{\cancel{(x-3)^n}\cdot(x-3)}{(2n+3)\cdot\cancel{2^n}\cdot 2} \cdot \frac{(2n+1)\cancel{2^n}}{\cancel{(x-3)^n}} \right|$$

$$\lim_{n\to\infty} \frac{2n+1}{4n+6} \cdot |x-3| < 1$$

$$2\cdot\frac{1}{2}|x-3| < 1 \cdot 2$$

$$|x-3| < 2 \quad \text{radius of convergence } \boxed{2}$$

c) $f(x) = \sum_{n=0}^{\infty} a_n (x-a)^n \quad = \quad \sum$ general term

$$f(4) = \sum_{n=0}^{\infty} \frac{(-1)^n (4-3)^n}{(2n+1)2^n}$$

1^n

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)2^n}$$

$f(4) = \frac{(-1)^4(4-3)^4}{(2\cdot4+1)(2^4)}$

$= \frac{1}{9\cdot16}$

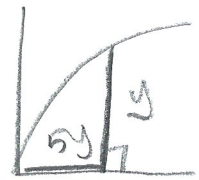
$= \frac{1}{144}$

$.007 < 0.01$

Alternating series test $\frac{1}{(2n+1)2^n} > \frac{1}{(2n+3)2^{n+1}}$ terms get smaller

$\lim_{n\to\infty} \frac{1}{(2n+1)2^n} = 0$ Thus it converges.

Therefore the error is less than the magnitude of the first omitted term



④ $y=\sqrt{8\sin\left(\frac{\pi x}{6}\right)}$

rectangle $A = b \times h$

a) $A = b \times h$

$= (5y)(y)$

$A = 5\left(\sqrt{8\sin\left(\frac{\pi x}{6}\right)}\right)\cdot\left(\sqrt{8\sin\left(\frac{\pi x}{6}\right)}\right)$

$= 5\cdot 8\sin\left(\frac{\pi x}{6}\right)$

$= 40\sin\left(\frac{\pi x}{6}\right)$

$\int_0^1 40\sin\left(\frac{\pi x}{6}\right)dx$

$u = \frac{\pi x}{6}$

$du = \frac{\pi}{6}dx$

$\frac{6}{\pi}du = dx$

$40\int_0^1 \sin\left(\frac{\pi x}{6}\right)dx$

$\frac{40\cdot 6}{\pi}\int_0^1 \sin(u)\,dx$

$-\frac{240}{\pi}\cos(u)\Big|_0^1$

$-\frac{240}{\pi}\cos\left(\frac{\pi x}{6}\right)\Big|_0^1$

$-\frac{240}{\pi}\left(\cos\left(\frac{\pi}{6}\right) - \cos(0)\right)$

$\boxed{-\frac{240}{\pi}\left(\frac{\sqrt{3}}{2} - 1\right)}$

b) circle πr^2 $r = x$

$$\int_0^2 \pi x^2 \, \Delta y$$

solve for x

$$y = \sqrt{8\sin\left(\frac{\pi x}{6}\right)}^{\,2}$$

$$\frac{y^2}{8} = \frac{8\sin\left(\frac{\pi x}{6}\right)}{8}$$

$$\sin^{-1}\frac{y^2}{8} = \sin^{-1}\sin\left(\frac{\pi x}{6}\right)$$

$$\frac{6}{\pi}\cdot\sin^{-1}\left(\frac{y^2}{8}\right) = \frac{\pi x}{6}\cdot\frac{6}{\pi}$$

$$x = \frac{6}{\pi}\sin^{-1}\left(\frac{y^2}{8}\right)$$

$$\boxed{\pi\int_0^2 \left(\frac{6}{\pi}\cdot\sin^{-1}\left(\frac{y^2}{8}\right)\right)dy}$$

5 a) volume of a cylinder is $V = \pi r^2 h$

$V = \pi(r^2 h)$ product rule for r and h $fg' + gf'$

$$\frac{dV}{dt} = \pi\left(r^2 \cdot 1 \frac{dh}{dt} + 2r \cdot h \frac{dr}{dt}\right)$$

$$\frac{dV}{dt} = \pi\left(r^2 \cdot \frac{dh}{dt} + 2rh \cdot \frac{dr}{dt}\right)$$

volume is constant not changing

$r = \frac{1}{2}$ in

$\frac{dh}{dt} = \frac{40ft}{sec} \times \frac{12in}{1ft} = \frac{480 in}{1 sec}$

$$\frac{0}{\pi} = \frac{\pi}{\pi}\left(\left(\frac{1}{2}\right)^2 \cdot 480 + 2 \cdot \left(\frac{1}{2}\right) \cdot 1200 \frac{dr}{dt}\right)$$

person is 100ft down

$h = 100 feet \times \frac{12 in}{1 ft}$

$h = 1200 in$

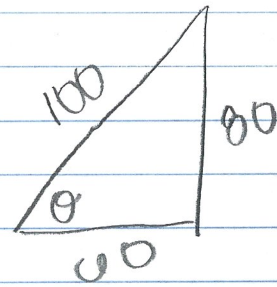
$$0 = \frac{1}{4} \cdot 480 + 1200 \frac{dr}{dt}$$

$-120 \quad -120$

$$\frac{-120}{1200} = \frac{1200}{1200}\frac{dr}{dt}$$

$$-\frac{1}{10} = \frac{dr}{dt} = \boxed{-\frac{1}{10} \text{ in/sec}}$$

b)



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$\leftarrow$

$\sec = \frac{H}{A}$

$\tan\theta = \frac{h}{60}$ ← since h is changing

$\frac{d}{dt}\tan\theta = \frac{d}{dt}\frac{h}{60}$ find $\frac{d\theta}{dt}$

$$\sec^2\theta \cdot 1\frac{d\theta}{dt} = \frac{1}{60}\cdot 1\frac{dh}{dt}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{60}\frac{dh}{dt}$$ $\frac{dh}{dt} = -40\text{ ft/sec}$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{60}\cdot(-40)$$

$$\left(\frac{100}{60}\right)^2\frac{d\theta}{dt} = -\frac{2}{3}$$

$$\frac{\cancel{100}}{\cancel{36}}\frac{d\theta}{dt} = -\frac{\cancel{2}^{\,1}}{\cancel{3}_{\,1}}\cdot\frac{\cancel{36}^{\,12}}{\cancel{100}_{\,50}}$$

$$\frac{d\theta}{dt} = -\frac{12}{50} \div 2 = \boxed{-\frac{6}{25}\text{ rad/sec}}$$

⑥ Interval $[-2,6]$. $F(x) = \int_1^x f(t)\,dt$

a) $F(-2) = \int_1^{-2} f(t)\,dt$

$= -\int_{-2}^{1} f(t)\,dt$ area under the curve

$= -[2(3)] = \boxed{-6}$

$F(6) = \int_1^6 f(t)\,dt$

$= \frac{1}{2}(1)(2) - \frac{1}{2}(4)(2)$

$= 1 - 4 = \boxed{-3}$

b) when $F(x) = 0$

$\int_1^1 f(t)\,dt$ = area from 1 to 1 is zero

when $\boxed{x = 1}$

look for when the area is the same above and below the x-axis

at $\boxed{x = 3}$

$x = 1$ to 3

$\frac{1}{2}(1)(2) - \frac{1}{2}(1)(2) = 0$

c) F is increasing when the area under the curve is positive

-2 to 2, when f is positive

$\boxed{-2 \le x \le 2}$

d) $F(x) = \int_1^x f(t)\,dt$

$F'(x) = f(x)$

max is when $F'(x) = f(x)$ changes from pos. to neg. at $x = 2$

max-value-plug in $x=2$ $\int_1^2 f(t)\,dt = \frac{1}{2}(1)(2) = \boxed{1}$

min is when $F'(x) = f(x)$ change from neg. to pos. Since that does not happen in our interval, we need to look at our endpoints

$x = -2$ and $x = 6$

from part (a)

$F(-2) = -6$ $F(6) = -3$

since $F(-2)$ is lower the min is at

$\boxed{x = -2, \text{ with min value at } -6}$

e) inflection points when $F''(x) = 0$ or changes sign

$F''(x) = f'(x)$ when it changes

at $\boxed{x = 3}$ negative slope to positive slope