BC Practice Exam #1

① $\lim_{x\to\infty} \frac{20x^2-13x+5}{5-4x^3} = \boxed{0}$ higher power on bottom

C

② $\lim_{h\to 0} \frac{\ln(2+h)-\ln 2}{h}$

plug in $\frac{\ln(2+0)-\ln 2}{0} = \frac{0}{0}$

L'Hopitals rule

$\frac{f'(x)}{g'(x)} = \lim_{h\to 0} \frac{\frac{1}{2+h}}{1} = \lim_{h\to 0} \frac{1}{2+h} = \frac{1}{2+0} = \boxed{\frac{1}{2}}$ C

③ $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{1}{\sqrt{1-t^2}}}{\frac{1}{\cancel{2}}(1-t^2)^{-1/2}\cdot -\cancel{2}t}$

$= \frac{\frac{1}{\sqrt{1-t^2}}}{\frac{-t}{\sqrt{1-t^2}}} = \frac{1}{\cancel{\sqrt{1-t^2}}} \cdot \frac{\cancel{\sqrt{1-t^2}}}{-t} = \boxed{\frac{-1}{t}}$

E

④ slope $[1,4)$

$(1,2)\ (4,6)$ $\frac{2-6}{1-4} = \frac{-4}{-3} = \boxed{\frac{4}{3}}$ B

⑤ $h(x) = g(f(x))$

Chain rule

$h'(x) = g'(f(x)) \cdot f'(x)$

$h'(3) = g'(f(3)) \cdot f'(3)$

$g'(4) \cdot 2$

$\frac{1}{2} \cdot 2 = \boxed{1} = B$

⑥ $\int_1^2 (3x-2)^3 dx$

$u = 3x-2$

$\frac{du}{3} = \frac{3dx}{3}$

$\frac{du}{3} = dx$

$\frac{1}{3}\int_1^2 u^3 du$

$\frac{1}{3}\frac{u^4}{4}\Big|_1^2$

$\frac{1}{12}(3x-2)^4\Big|_1^2$

$\frac{1}{12}(3(2)-2)^4 - \frac{1}{12}(3(1)-2)^4$

$\frac{1}{12}(4)^4 - \frac{1}{12}(1)^4$

$\frac{1}{12} \cdot 256 - \frac{1}{12} = \frac{255 \div 3}{12 \div 3} = \boxed{\frac{85}{4}}$ D

$64 \times 4 = 256$

$255 \div 3 = 85$; 24, 15

$fg' + gf'$

7. $y = \frac{x-3}{2-5x} = (x-3)(2-5x)^{-1}$

$= (x-3)\cdot -1(2-5x)^{-2} \cdot -5 + (2-5x)^{-1} \cdot 1$

$\frac{5(x-3)}{(2-5x)^2} + \frac{1}{(2-5x)} \cdot \frac{(2-5x)}{(2-5x)}$

$\frac{5x-15+2-5x}{(2-5x)^2} = \boxed{\frac{-13}{(2-5x)^2}}$ E

8. $f(x) = xe^{-x}$ $\quad fg' + gf'$

$f'(x) = x \cdot e^{-x} \cdot -1 + e^{-x} \cdot 1$

$= -xe^{-x} + e^{-x} = e^{-x}(1-x) = 0$

$= \cancel{e^{-x} = 0}$ $\quad 1-x=0$

$-x=-1$

$\boxed{x=1}$

$x=0$ $\quad x=2$

plug back in

$f(1) = 1 \cdot e^{-1} = e^{-1} = \boxed{\frac{1}{e}}$ A

⑨ when $y=0$ slope is undefined

$\frac{dy}{dx} = \frac{5}{y}$ or $\cancel{\frac{dy}{dx} = \frac{x}{y}}$

$\boxed{A}$ 3rd Q $\frac{-x}{-y}$ = positive slope its neg.

⑩ make slope of graph

~~A) 2<t<5~~ slope = 0

B) 5<t<8 (5,2) (8,-4

$\frac{2--4}{5-8} = \frac{6}{-3} = -2$

c) t=6 same m = -2

~~D) t=8~~ m=0

E) (8,-4) (9,0) $\frac{-4-0}{8-9} = \frac{-4}{-1} = \boxed{4}$ E

⑪ Since velocity is positive from 0→6 its moving to the right when negative it moves left

Thus $\boxed{t=6}$ C

(12) $x = 2\sin\theta$ $\quad \int_0^2 \frac{x^2\,dx}{\sqrt{4-x^2}}$

$\frac{2}{2} = \frac{2\sin\theta}{2}$

$\sin^{-1} 1 = \sin^{-1}\sin\theta$

$\sin^{-1} 1 = \theta$

$\theta = \pi/2$

$\int_0^{\pi/2} \qquad dx$

$= \frac{(2\sin\theta)^2}{\sqrt{4-(2\sin\theta)^2}} \cdot dx$

$\frac{0}{2} = \frac{\cancel{2}\sin\theta}{\cancel{2}}$

$0 = \sin\theta$

$\sin^{-1} 0 = \theta$

$\theta = 0$

$= \frac{4\sin^2\theta}{\sqrt{4-4\sin^2\theta}}$

$= \frac{4\sin^2\theta}{\sqrt{4(1-\sin^2\theta)}}$

$= \frac{\cancel{4}^{2}\sin^2\theta}{\cancel{2}\sqrt{1-\sin^2\theta}}$

$\frac{d}{d}$

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$= \frac{2\sin^2\theta}{\sqrt{\cos^2\theta}}$

$= \frac{2\sin^2\theta}{\cos\theta} =$

$\int_0^{\pi/2} \frac{2\sin^2\theta}{\cancel{\cos\theta}} \cdot 2\cancel{\cos\theta}\,d\theta$

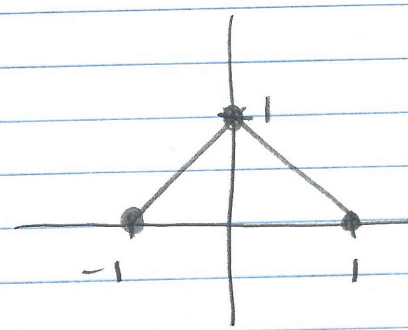
$\boxed{\int_0^{\pi/2} 4\sin^2\theta\,d\theta}$ B

(13) $\int_{-1}^{1} (1-|x|)\,dx$

$1-|x|$

x	y
1	0
0	1
-1	0

area under the curve



$\frac{1}{2}(1)(1) + \frac{1}{2}(1)(1)$

$\frac{1}{2} + \frac{1}{2} = \boxed{1}$ C

(14) $\lim_{x\to\infty} x^{1/x}$ $\quad \infty^0$ undeterminate form

$y = x^{1/x}$

$\ln y = \ln x^{1/x}$

$\ln y = \frac{1}{x}\ln x \quad = \frac{\ln x}{x}$

L'Hopitals Rule

$\frac{f'(x)}{g'(x)} \lim_{n\to\infty} \frac{\frac{1}{x}}{1} = \lim_{n\to\infty} \frac{1}{x} = 0$

then $e^0 = \boxed{1}$ B

$\begin{array}{r} 1.73 \\ -1.39 \\ \hline .34 \end{array}$

(15) Slope at 2.1

$(2.0, 1.39) \quad (2.2, 1.73)$

$\frac{1.73-1.39}{2.2-2.0} = \frac{.34}{.2} \quad \frac{3.4}{2}$

$\boxed{1.7}$ D