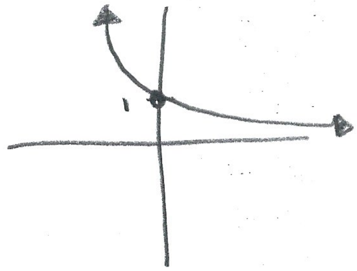
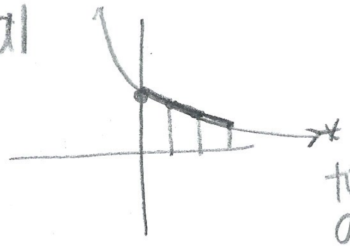
BC #1

(16) graph of e^{-x}



A = actual area

trapezoidal

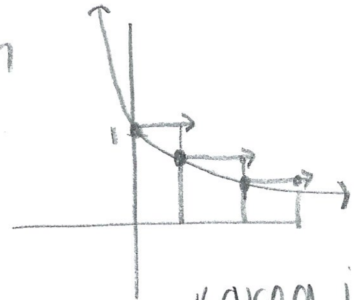


$A \leq T$

*very close to actual area or a little above

- make intervals
- draw trapezoids

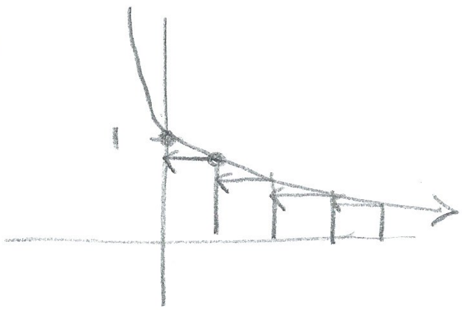
left sum
(opposite)



- make intervals
- at each interval point draw arrow to the right, make a rectangle

*area is greater than actual area

right sum
(opposite)



- make intervals
- at each interval point draw an arrow to the left, make a rectangle.

*area is smaller than the actual area

$\boxed{R \leq A \leq T \leq L}$ A

BC #1

(17) $\frac{dy}{dx} = y\tan x$ $y=3$ when $x=0$ $\Rightarrow (0,3)$

*write a $y=$ equation

* separate dy and dx, then integrate

$$\int \frac{dy}{y} = \int \tan x \, dx$$

$\ln|y| = -\ln|\cos x| + c$

Find c, plug in $(\underset{x}{0}, \underset{y}{3})$

$\ln 3 = -\ln|\cos(0)| + c$

$\ln 3 = -\ln 1 + c$

$\ln 3 = -0 + c$

$c = \ln 3$

so we get

$\ln|y| = -\ln|\cos x| + \ln 3$

*now we need to solve for y

$\ln|y| = -\ln|\cos x| + \ln 3$

$\ln|y| + \ln|\cos x| = \ln 3$

if I'm adding 2 ln's I can multiply them

$e^{\ln|y(\cos x)|} = e^{\ln 3}$

*raise each side by e to get rid of the ln

$\frac{y(\cancel{\cos x})}{\cos x} = \frac{3}{\cos x}$

$y = \frac{3}{\cos x}$

we need to find y when $x = \pi/3$

*plug in $\frac{\pi}{3}$ for x

$y = \frac{3}{\cos\frac{\pi}{3}} = \frac{3}{\frac{1}{2}}$

$= 3 \cdot \frac{2}{1}$

$= \boxed{6}$ E

BC #1

(18) $\int_0^6 f(x-1)\,dx =$

* I changed all of my answers into y's so I'm comparing two different letters

A) $\int_{-1}^{7} f(y)\,dy$ B) $\int_{-1}^{5} f(y)\,dy$ C) $\int_{-1}^{5} f(y+1)\,dy$

D) $\int_{1}^{5} f(y)\,dy$ E) $\int_{1}^{7} f(y)\,dy$

we know that $y = x-1$

since 0 and 6 are my x limits, we need to change them into terms of y using $y=x-1$

$x=0$ $x=6$

$y=0-1$ $y=6-1$

$y=-1$ $y=5$

Thus $\boxed{\int_{-1}^{5} f(y)\,dy}$

B

and since $y=x-1$

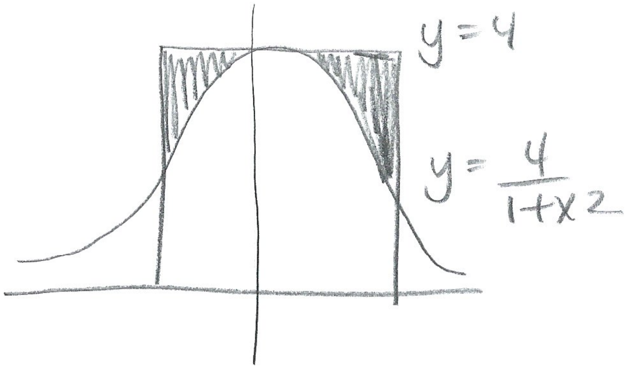
Find dy ~~of~~ $y=x-1$

$dy = 1dx$

$dy = dx$

BC #1

19 $y = \dfrac{4}{1+x^2}$



Since the area on each side is the same, we can calculate the area from 0 to 1 and then double it :)

$$2\int_0^1 (\text{top} - \text{bottom})\,dx$$

$$2\int_0^1 \left(4 - \frac{4}{1+x^2}\right) dx$$

$$2\int_0^1 \left(4 - 4\cdot\left(\frac{1}{1+x^2}\right)\right) dx$$

$$4x - 4\cdot \tan^{-1}x \Big|_0^1$$

plug in 1 and 0

$$\left[4(1) - 4\tan^{-1}(1)\right] - \left[\cancel{4(0)}^{\,0} - 4\tan^{-1}(0)\right]$$

tan ? = 1 tan ? = 0

$$\left[4 - \cancel{4}\cdot\frac{\pi}{\cancel{4}}\right] - \left[0 - \cancel{4\cdot 0}^{\,0}\right]$$

$$2(4-\pi) = \boxed{8-2\pi} \quad B$$

BC #1

(20) slope of the curve $r = \cos 2\theta$ at $\theta = \frac{\pi}{6}$

$*$ slope $= \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $\Big\{$ $*$ polar coordinate are written as $x = r\cos\theta$ and $y = r\sin\theta$

$*$ we need to find $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ using the product rule with the above polar coordinates.

$fg' + gf'$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{r \cdot \cos\theta + \sin\theta \cdot 1\, dr/d\theta}{r \cdot -\sin\theta + \cos\theta \cdot 1\, dr/d\theta}$$

$\rightarrow$ since I took the derivative of r with respect to θ

$*$ we know that $r = \cos 2\theta$ from the given problem

$*$ we can find $\frac{dr}{d\theta}$ by taking the derivative of $r = \cos 2\theta$

$$r = \cos 2\theta$$

$$\frac{dr}{d\theta} = -\sin(2\theta) \cdot 2 = -2\sin(2\theta)$$

$*$ now plug in both

$$= \frac{\cos(2\theta) \cdot \cos\theta + \sin\theta \cdot -2\sin(2\theta)}{\cos(2\theta) \cdot -\sin\theta + \cos\theta \cdot -2\sin(2\theta)}$$

$$= \frac{\cos(2\theta) \cdot \cos\theta - 2\sin\theta \cdot \sin(2\theta)}{-\cos(2\theta) \cdot \sin\theta - 2\cos\theta \cdot \sin(2\theta)}$$

$*$ plug in $\theta = \frac{\pi}{6}$

$\frac{\pi}{6}\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\frac{\pi}{3}\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$$= \frac{\cos(\pi/3) \cdot \cos(\pi/6) - 2\sin(\pi/6) \cdot \sin(\pi/3)}{-\cos(\pi/3) \cdot \sin(\pi/6) - 2\cos(\pi/6) \cdot \sin(\pi/3)}$$

$$\frac{\frac{1}{2} \cdot \frac{\sqrt{3}}{2} - 2 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2}}{-\frac{1}{2} \cdot \frac{1}{2} - 2\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2}}$$

$$= \frac{\frac{\sqrt{3}}{4} - \frac{2\sqrt{3}}{4}}{-\frac{1}{4} - \frac{6}{4}} = \frac{\frac{-1\sqrt{3}}{4}}{\frac{-7}{4}}$$

$$\frac{-1\sqrt{3}}{4} \cdot \frac{4}{-7} = \boxed{\frac{\sqrt{3}}{7}}$$

BC #1

21

21. A particle moves along a line with velocity, in feet per second, $v = t^2 - t$. The total distance, in feet, traveled from $t = 0$ to $t = 2$ equals

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 2 (D) 1 (E) $\frac{4}{3}$

* we want to see if the particle stops on the interval $[0,2]$
Thus, we look for when the $v(t)=0$

$v(t) = t^2 - t = 0$

$t(t-1) = 0$

$\boxed{t=0}$ $t-1=0$

$\boxed{t=1}$

* lets check what direction the particle is heading, check test points into $t(t-1)$

$t=0$ $t=1$ $t=2$

$t=\frac{1}{2}$ $t=1\frac{1}{2}$

$-$ $+$

moving left $t=0$ to $t=1$

moving right $t=1$ to $t=2$

* To look at distance traveled from velocity, we need to integrate.

$$-\int_0^1 (t^2-t)\,dt \quad \int_1^2 (t^2-t)\,dt$$

since we are moving left it is negative distance, we want to add up the total distance.

$$-\left[\frac{t^3}{3}-\frac{t^2}{2}\right]\Big|_0^1 + \left[\frac{t^3}{3}-\frac{t^2}{2}\right]\Big|_1^2$$

$$\left[-\left(\frac{1}{3}-\frac{1}{2}\right) - -\left(0-0\right)\right] + \left[\left(\frac{8}{3}-\frac{4}{2}\right)-\left(\frac{1}{3}-\frac{1}{2}\right)\right]$$

$$-\frac{1}{3}+\frac{1}{2}+\frac{8}{3}-\frac{4}{2}-\frac{1}{3}+\frac{1}{2} = \frac{6}{3}-1 = 2-1$$

$\frac{6}{3}$ $\frac{2}{2}$

$= \boxed{1}$ D

BC #1

22

22. The general solution of the differential equation $\frac{dy}{dx} = \frac{1-2x}{y}$ is a family of

(A) straight lines (B) circles (C) hyperbolas
(D) parabolas (E) ellipses

* First separate dy and dx to each side

$$\frac{dy}{dx} = \frac{1-2x}{y}$$

$$\int y\,dy = \int (1-2x)\,dx \qquad \text{* Integrate}$$

$$\frac{y^2}{2} = x - \frac{2x^2}{2} + C$$

$$2\left(\frac{y^2}{2} = x - x^2 + C\right)$$

$$y^2 = 2x - 2x^2 + 2C$$

$$y^2 + 2x^2 - 2x - 2C = 0$$

$$x^2 + y^2 = \boxed{\text{ellipse}} \quad E$$

BC #1

23

23. The curve $x^3+x\tan y=27$ passes through (3,0). Use local linear approximation to estimate the value of y at $x=3.1$. The value is

(A) -2.7 (B) -0.9 (C) 0 (D) 0.1 (E) 3.0

*we want to write an equation of a line $(y=mx+b)$ so first we need to find $m=dy/dx$

Take the derivative of $x^3+\underbrace{x\cdot\tan y}_{\text{product rule}}=27$ $fg'+gf'$

$$3x^2dx+x\cdot(\sec^2y)\cdot 1\cdot dy+\tan y\cdot 1dx=0$$

$$3x^2dx+x\sec^2y\,dy+\tan y\,dx=0$$

*get dy to one side and dx to the other

$$x\sec^2y\,dy=-3x^2dx-\tan y\,dx$$

$$x\sec^2y\,dy=(-3x^2-\tan y)dx$$

$$\frac{dy}{dx}=\frac{-3x^2-\tan y}{x\sec^2y}$$ plug in (3,0) to get the slope

$\sec y=\frac{1}{\cos y}$ $$\frac{dy}{dx}=\frac{-3(3)^2-\tan(0)}{3\sec^2(0)}=\frac{-27-0}{3\cdot 1}=\frac{-27}{3}=$$

$m=-9$

*write an equation of a line

$m=-9$ (3,0)

$y=mx+b$

$0=-9\cdot 3+b$

$0=-27+b$

$b=27$

$y=-9x+27$

Now plug in $x=3.1$

$y=-9\cdot(3.1)+27$

$=-27.9+27$

$=\boxed{-0.9}$ B

BC #1

24

24. $\int x \cos x \, dx =$

(A) $x \sin x + \cos x + C$ (B) $x \sin x - \cos x + C$

(C) $\frac{x^2}{2} \sin x + C$ (D) $\frac{1}{2} \sin x^2 + C$ (E) none of these

$\int x \cdot \cos x \, dx$

tic-tac-toe method

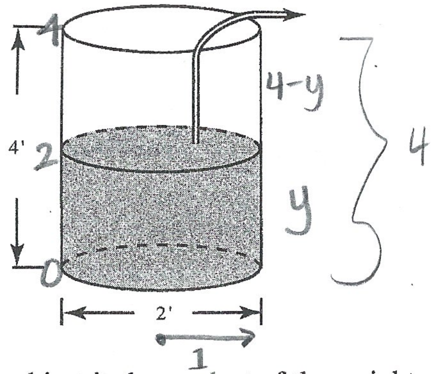
derivative		integrate
x		$\cos x$
	$+$	
1		$\sin x$
	$-$	
0		$-\cos x$

$x \sin x + +\cos x + C$

$\boxed{x \sin x + \cos x + C}$ A

BC #1

25



25. The work done in lifting an object is the product of the weight of the object and the distance it is moved. A cylindrical barrel 2 feet in diameter and 4 feet high is half-full of oil weighing 50 pounds per cubic foot. How much work is done, in foot-pounds, in pumping the oil to the top of the tank?

(A) 100π (B) 200π (C) 300π (D) 400π (E) 1200π

work = (weight of the oil) • (distance the oil is raised)

50 x volume

$$= (50 \cdot \pi \cdot 1^2 \cdot \Delta y) \cdot (4-y)$$

volume of a cylinder $\pi r^2 h$

since the volume changes constantly (the change in y)

$$= \int_0^2 50\pi\,(4-y)\,dy$$

since its half full

$$= 50\pi \int_0^2 (4-y)\,dy$$

$$50\pi \left(4y - \frac{y^2}{2}\right)\Big|_0^2$$

$$50\pi \left[\left(4(2) - \frac{2^2}{2}\right) - \left(\cancel{4(0)} - \cancel{\frac{0^2}{2}}\right)\right]$$

$$50\pi(8-2) = 50\pi(6) = \boxed{300\pi}$$ C

BC #1

26

26. The coefficient of the $(x-8)^2$ term in the Taylor polynomial for $y = x^{2/3}$ centered at $x = 8$ is

(A) $-\frac{1}{144}$ (B) $-\frac{1}{72}$ (C) $-\frac{1}{9}$ (D) $\frac{1}{144}$ (E) $\frac{1}{6}$

* given equation $y=x^{2/3}$ centered at $x=8$
we want to know the coefficient of the 3rd term

1st term $f(8) = 8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = \boxed{4}$

2nd term $\frac{f'(8)(x-8)}{1!}$

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

$f'(8) = \frac{2}{3\sqrt[3]{8}} = \frac{2}{3\cdot 2} = \frac{2}{6} = \frac{1}{3}$

so $\frac{1}{3}\frac{(x-8)}{1!} = \boxed{\frac{1}{3}(x-8)}$

3rd term $\frac{f''(8)(x-8)^2}{2!}$

$f''(x) = \frac{2}{3}\cdot\frac{-1}{3}x^{-4/3} = \frac{-2}{9(\sqrt[3]{x})^4}$

$f''(8) = \frac{-2}{9(\sqrt[3]{8})^4} = \frac{-2}{9\cdot 24} = \frac{-1}{72}$

so $\frac{-1}{72}\frac{(x-8)^2}{2!} = \frac{-1}{72\cdot 2}(x-8)^2$

$2\cdot 1$

$= \boxed{\frac{-1}{144}}(x-8)^2$

A

BC #1

27

27. If $f'(x) = h(x)$ and $g(x) = x^3$, then $\frac{d}{dx} f(g(x)) =$

(A) $h(x^3)$ (B) $3x^2h(x)$ (C) $h'(x)$ (D) $3x^2h(x^3)$ (E) $x^3h(x^3)$

$\frac{d}{dx} f(g(x))$ $\rightarrow$ is basically the chain rule

remember think about how you would take the derivative of $\cos(2x)$

$= f'(g(x)) \cdot g'(x)$

we know that $g(x) = x^3$

thus, $g'(x) = 3x^2$ (just take the derivative)

lets plug that in

$= f'(x^3) \cdot 3x^2$

we also know that $f'(x) = h(x)$

so we can replace the f' with an h

$= h(x^3) \cdot 3x^2$

$= \boxed{3x^2 h(x^3)}$ D

BC #1

28

28. $\int_0^{\infty} e^{-x/2}\,dx =$

(A) $-\infty$ (B) -2 (C) 1 (D) 2 (E) ∞

This is an improper integral since $\int^{\infty}$ ←

*remember replace ∞ with b, and put $\lim_{b\to\infty}$ in front

$\lim_{b\to\infty} \int_0^b e^{-x/2}\,dx$ now integrate

let $u = \frac{-x}{2}$

$du = -\frac{1}{2}dx$

$-2du = dx$

$\lim_{b\to\infty} -2\int_0^b e^{u}\,du$

$\lim_{b\to\infty} -2e^{-x/2}\Big|_0^b$

$\lim_{b\to\infty} -2e^{-b/2} + +2e^{0}$

$\lim_{b\to\infty} \cancel{-2e^{-b/2}}^{\,0} + 2$

$\boxed{2}$ D

graph of $e^{-x/2}$

as it approaches ∞, it gets close to zero