Unit #1: Functions — practice exercises

#1) $f(x) = x^3 - 2x - 1$

$f(-2) = (-2)^3 - 2(-2) - 1$
$= -8 + 4 - 1 = \boxed{-5}$ C

#2) $f(x) = \dfrac{x-1}{x^2+1}$ Domain $x^2+1 = 0$
$x^2 = -1$
not possible

Thus, domain is all reals
since the denominator cant equal 0

E

#3) $f(x) = \dfrac{\sqrt{x-2}}{x^2-x} = \dfrac{\sqrt{x-2}}{x(x-1) = 0}$ $x(x-1) = 0$
$\boxed{x \neq 0 \quad x \neq 1}$

and $\sqrt{x-2} \geq 0$
$x - 2 \geq 0$
$x \geq 2$

since $x \geq 2$ and $x \neq 0$ and $x \neq 1$

only need $\boxed{x \geq 2}$ D

#4) $g(f(x)) = g(x^3 - 3x^2 - 2x + 5) = 2$

$g(\quad) = \boxed{2}$ E

#5) $f(g(x)) = f(2) = 2^3 - 3 \cdot 2^{\overset{4}{2}} - 2(2) + 5$

$= 8 - 12 - 4 + 5$

$= 13 - 16 = \boxed{-3}$ D

#6) $f(x) = x^3 + Ax^2 + Bx - 3$ $\qquad f(1) = 4 \quad f(-1) = -6$

$f(1) = 1^3 + A(1)^2 + B(1) - 3 = 4$

$1 + A + B - 3 = 4$

$A + B - 2 = 4$

$A + B = 6$

$f(-1) = (-1)^3 + A(-1)^2 + B(-1) - 3 = -6$

$-1 + A - B - 3 = -6$

$A - B - 4 = -6$
$\quad\;\; +4 \quad +4$

$A - B = -2$

$A + \cancel{B} = 6$

$A - \cancel{B} = -2$

$2A = 4$

$\boxed{A = 2}$

$A + B = 6$

$2 + B = 6$

$\boxed{B = 4}$

$2A + B = 2(2) + 4 = 4 + 4 = \boxed{8}$

B

#7) odd function → symmetric to origin

$f(-x) = -f(x)$

~~A)~~ $f(-x) = \frac{-x-1}{-x} = \frac{x+1}{x} \neq -f(x)$

B) $f(-x) = 2(-x)^4+1 = 2x^4+1 \neq -f(x)$

C) $f(-x) = (-x)^3+2(-x) = -x^3-2x = -(x^3+2x)$
$= -f(x)$

C

ex: $y = \sin x$ symmetric but not one-to-one

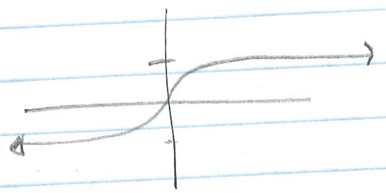
#8) C → symmetric about origin does not make it a function   pg# 609

In order for g to have an inverse, it must be one-to-one $ax+b$ is a function

A function that is increasing/decreasing is one-to-one, thus has an inverse

#9) $f(x) = \sin(\arctan x)$ $\tan^{-1}$

$\sin x$ ← arctan x $-\frac{\pi}{2} < x < \frac{\pi}{2}$



Since the range of a sin function is between −1 and 1 no matter what you plug in the range will still be the same

Since $\tan = \frac{y}{x}$ we can use the points (0,1) and (0,−1)

since $x \neq 0$.

$\sin -\frac{\pi}{2} < \sin \tan^{-1} x < \sin \frac{\pi}{2}$

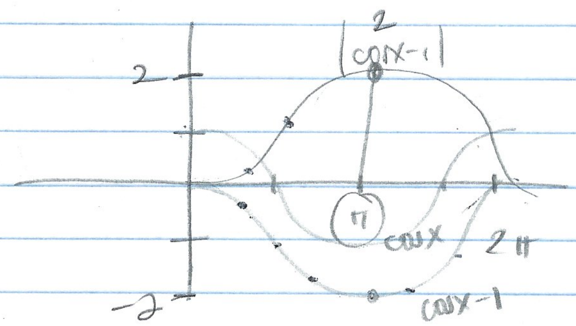
B $\boxed{y \mid -1 < y < 1}$

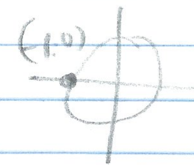
$\boxed{-1 < \sin(\tan^{-1} x) < 1}$

#10) $g(x) = |\cos x - 1|$

E $\boxed{\pi}$

$\boxed{\text{Chart}}$





$|\cos x - 1| = 2$

$\cos x - \cancel{1} = 2$ $\quad$ $\cos x \cancel{-1} = -2$

$+1 \quad +1$ $\quad$ $+1 \quad +1$

$\cancel{\cos x = 3}$ $\quad$ $\cos x = -1$

$\cos x - \cancel{1} = -2$

$+1 \quad +1$

$\cos x = -1$

$\cos(\pi) = -1$

#11) odd means $f(-x) = -f(x)$

~~a)~~ $\sin(-x) = -\sin x = -f(x)$

~~b)~~ $\sin 2x = \sin(2)(-x) = \sin(-2x) = -\sin 2x = -f(x)$

c) $x^3 + 1 = (-x)^3 + 1 = -x^3 + 1 \neq -f(x)$

C

#12) roots of equation $f(x) = 0$ are 1, and -2

$(x-1)(x+2)$

$\downarrow$

$f(2x) = x^2 + x - 2 = (2x)^2 + 2x - 2$

$4x^2 + 2x - 2 = 0$

$2(2x^2 + x - 1)$

$2(x+1)(2x-1) = 0$

$\boxed{x = -1 \quad x = \frac{1}{2}}$

B

x $\quad$ $2x$ $\quad$ -2 $\quad$ $2x$

1 $\quad$ 2 $\quad$ -1

1

#13) $f(x) = x^3 + 4x^2 + 4x = 0$

$x(x^2 + 4x + 4)$

4

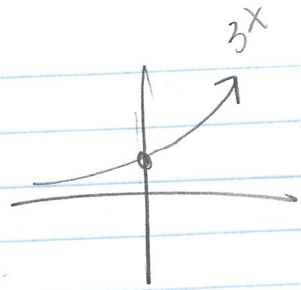
2 2

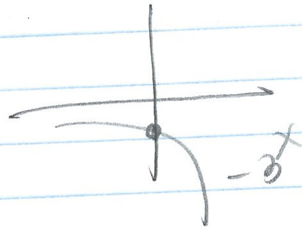
4

$x(x+2)^2 = 0$

$\boxed{x = 0,\ x = -2}$

B





#14) $y = x + 2$ and $y^2 = 4x$

solve the system

$y^2 = 4x$

$(x+2)^2 = 4x$

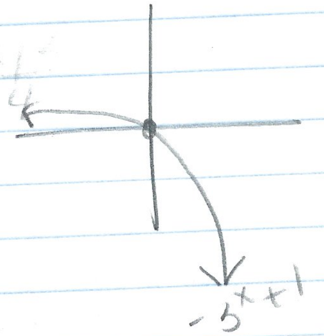
$x^2 + 4x + 4 = 4x$

$x^2 + 4 = 0$

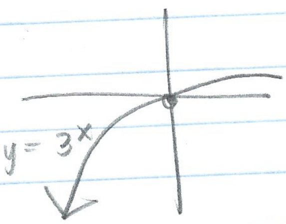
$\sqrt{x^2} = \sqrt{-4}$

not possible

E



y-axis

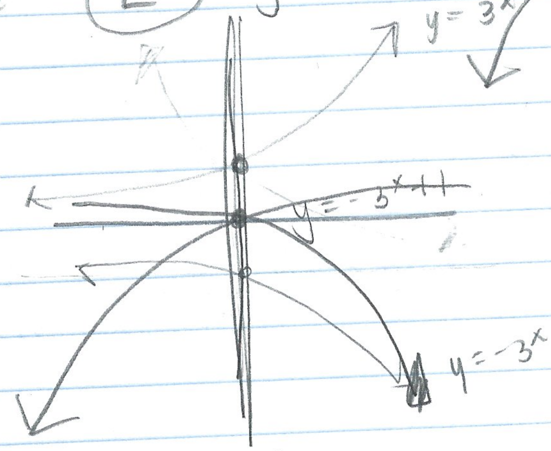


#15) $f(x) = 1 - 3^x$

$= -3^x + 1$

reflect ~~x~~ y-axis

shift up 1



want the same but reflect the y-axis

$y = 3^{-x}$

reflect y-axis

so $\boxed{y = -3^{-x} + 1}$

A

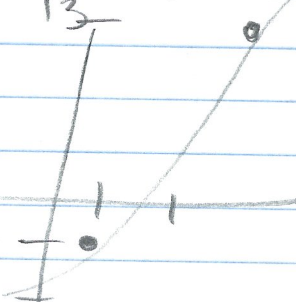
#16) $f(g(x)) = \boxed{x}$ B

if f and g are inverses

#17) $f(x)=2x^3+x-5$

x	y
-2	
-1	-8
0	-5
1	-1
2	13

since $f(1) = -1$ and $f(2)=13$ and also since the function

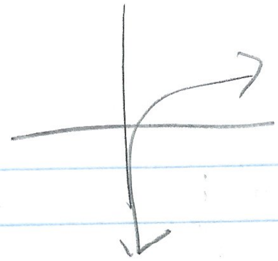


is continuous, it must cross the x-axis thus giving us a root since $f(1) = -1$ negative and $f(2) = 13$ positive it must cross between $\boxed{1 \text{ and } 2}$ D

#18) $f(x) = \sin \frac{2\pi}{3} x$

K

$$\text{period} = \frac{2\pi}{K} = \frac{2\pi}{2\pi/3} = \cancel{2\pi \cdot \frac{3}{2\pi}} = \boxed{3}$$ D



#19) $y = \ln(\cos x)$

since the range of $\cos x$ is $-1 \le \cos x \le 1$

now every the doman of ln must be positive
thus $\ln x$ where $x > 0$
thus the domain for the graph is
$0 < \cos x \le 1$

so $\ln 0 < \ln(\cos x) \le \ln 1$

$-\infty < \ln(\cos x) \le 0$

so range is $(-\infty, 0]$ or $\boxed{-\infty < y \le 0}$ A

#20) $\log_b(3^b) = \frac{b}{2}$

we can write $\frac{\cancel{b}\,\log_b 3}{\cancel{b}} = \frac{\cancel{b}}{2\,\cancel{b}}$

$\log_b 3 = \frac{1}{2}$

$b^{\frac{1}{2}} = 3$

$\sqrt{b}^{\,2} = 3^2$

$\boxed{b = 9}$ E

#21) $f(x) = x^3 + 2$ Find $f^{-1}(x)$

$y = x^3 + 2$

switch x and y

$x = y^3 + 2$

solve for y again

$\sqrt[3]{y^3} = \sqrt[3]{x-2}$

$y = \sqrt[3]{x-2}$ $f^{-1}(x) = \boxed{\sqrt[3]{x-2}}$ E

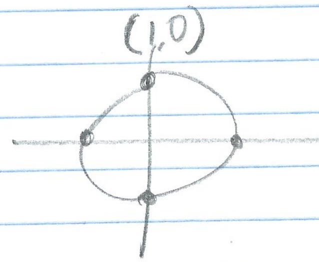
#22) $f(x) = \dfrac{x^2(x^3 - 2x^2) - 1(x+2)}{x-2 \quad\quad x-2} = 0$

$(x^2-1)(x-2) = 0$

$(x+1)(x-1)(x-2) = 0$

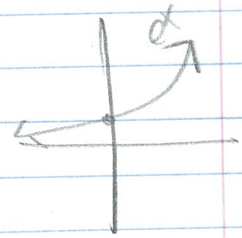
$\boxed{x = -1, 1, 2}$ D

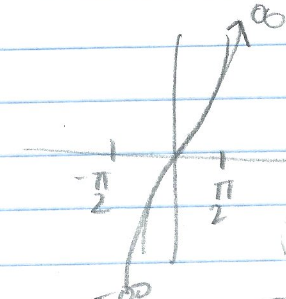
#23) domain $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ $f(x) = e^{\tan x}$



since $-\frac{\pi}{2} < x < \frac{\pi}{2}$

$\tan -\frac{\pi}{2} < \tan x < \tan \frac{\pi}{2}$



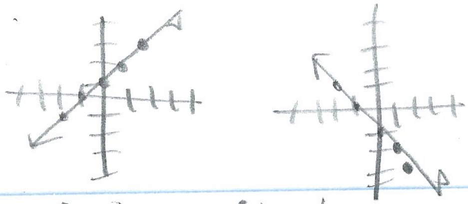


$-\infty < \tan x < \infty$

$e^{-\infty} < e^{\tan x} < e^{\infty}$

$0 < e^{\tan x} < \infty$, thus $\boxed{0 < y < \infty}$ B

does not include zero.



#24) y=f(x) reflection in x-axis

*opposite of y-coordinate

(A) $y = -f(x)$

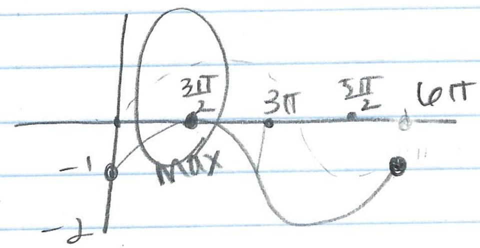
change the sign of y-coordinate reflecting over x-axis.

$y = x+1$

x	y
2	3
1	2
0	1
-1	0
-2	-1

x	y
2	-3
1	-2
0	-1
-1	0
-2	1

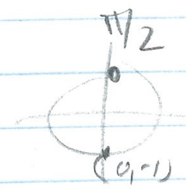
#25) $f(x) = \sin\left(\frac{x}{3}\right) - 1$



period $= \frac{2\pi}{k} = \frac{2\pi}{\frac{1}{3}}$

$= 2\pi \cdot \frac{3}{1} = 6\pi$

$\sin\frac{x}{3} - 1 = 0$



$\sin\frac{x}{3} = 1$

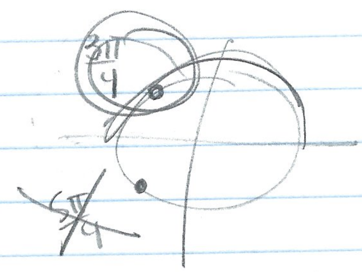
$\frac{3\pi}{2}$ (C)

$\therefore \sin\left(\frac{\pi}{2}\right) = 1$

$\frac{x}{3} = \frac{\pi}{2}$

$\boxed{x = \frac{3\pi}{2}}$

#26) $\tan\left(\arccos\left(-\frac{\sqrt{2}}{2}\right)\right) =$



$\tan\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$ range of $0 \le y \le \pi$

where does $\cos\boxed{} = -\frac{\sqrt{2}}{2}$

$\tan\left(\frac{3\pi}{4}\right) = \frac{y}{x} = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \boxed{-1}$ (A)

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

#27) If $f(x) = 2e^{-x}$

$y = 2e^{-x}$ switch x and y

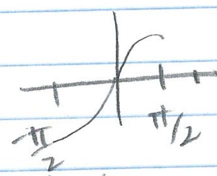
$\frac{x}{2} = \frac{2e^{-y}}{2}$ solve for y

$\ln \frac{x}{2} = \ln e^{-y}$

$\ln \frac{x}{2} = -y$

$y = -\ln \frac{x}{2} = \ln\left(\frac{x}{2}\right)^{-1}$

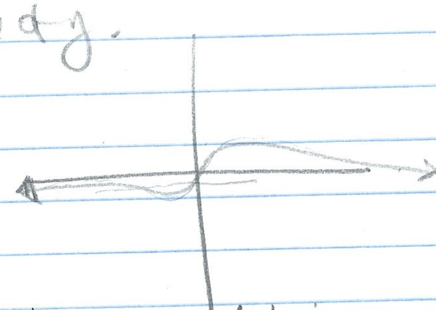
$f^{-1}(x) = \boxed{\ln\left(\frac{2}{x}\right)}$ A



#28) To have an inverse, the function must be one-to-one. → each x has one matching value of y.

C

$\frac{x}{x^2+1}$



horizontal line test fails

$\frac{x}{x^2+1} = \frac{1}{4}$

$x^2+1 = 4x$

$x^2 - 4x + 1 = 0$

solve we would get 2 answers for x thus not one-to-one

$\frac{4 \pm \sqrt{16 - 4(1)(1)}}{2(1)}$

$\frac{4 \pm \sqrt{16-4}}{2}$

$\frac{4 \pm \sqrt{12}}{2}$

$\frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$

#29) $f(x) = \ln x$ $\quad g(x) = 9 - x^2$

$f(g(x))$

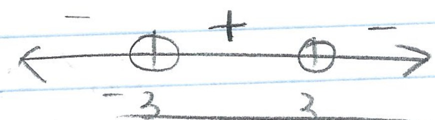
$\ln(9 - x^2)$

Since the domain for $\ln x$ is $x > 0$

thus $9 - x^2 > 0$

$(3 - x)(3 + x) > 0$

$x = 3 \quad x = -3$



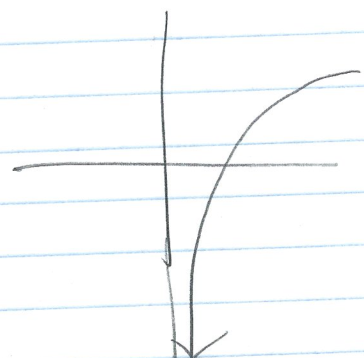
$x < 3$

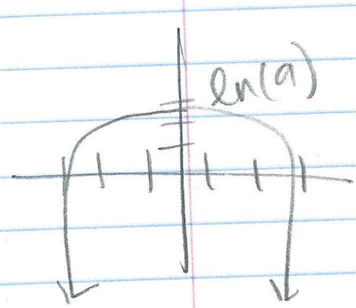
$x > -3$

$-3 < x < 3$

D $\boxed{|x| < 3}$ $\longleftrightarrow$ $\boxed{-3 < x < 3}$

#30) Since the domain is $\ln(9 - x^2)$ is $-3 < x < 3$





x	y
-3	$\ln 0$ $-\infty$
0	$\ln 9$ → biggest #
3	$\ln 0$ $-\infty$

thus $\ln(9 - x^2)$ will have a range of every # less than $\ln 9$

$\boxed{y \leq \ln 9}$ C

#31) $x(t) = t^2+3$ $y(t) = t^2+4$

$x = t^2+3$ $y = t^2+4$

$\downarrow$ plug into

$t^2 = x-3$

$y = (x-3)+4$

$\boxed{y = x+1}$ a line A

#32) $x(t) = \cos^2(t)$ $y(t) = 2\sin t$

$x = \cos^2(t)$ $y = 2\sin t$

$x = \cos^2(t)$ $\left(\frac{y}{2}\right)^2 = \sin^2 t$

$x = \cos^2(t)$ $\frac{y^2}{4} = \sin^2 t$

since

$\cos^2 t + \sin^2 t = 1$

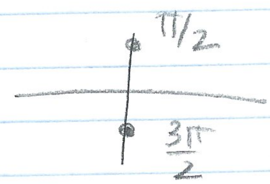
$\left(x + \frac{y^2}{4} = 1\right) 4$

$\boxed{4x + y^2 = 4}$ D

#33) $r = 2\cos(5\theta)$ origin when $r=0$

$$\frac{2\cos(5\theta)}{2} = \frac{0}{2}$$

$$\cos^{-1}\cos(5\theta) = \cos^{-1} 0$$



$$5\theta = \frac{\pi}{2} \text{ and } \frac{3\pi}{2}$$

$5\theta = \frac{\pi}{2}$ $5\theta = \frac{3\pi}{2}$

$\boxed{\theta = \frac{\pi}{10}}$ D $\theta = \frac{3\pi}{10}$

smaller

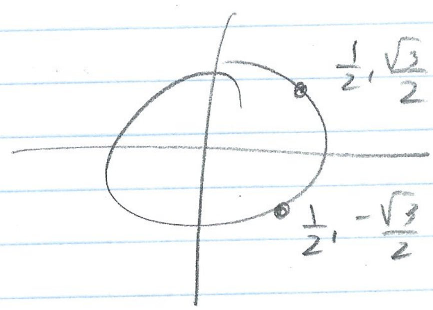
#34) $r = 3$ and $r = 2 + 2\cos\theta$

intersect = set them equal

$$2 + 2\cos\theta = 3$$
$$-2 \qquad\qquad -2$$

$$\frac{2\cos\theta}{2} = \frac{1}{2}$$

$$\cos\theta = \frac{1}{2}$$



$\boxed{\frac{\pi}{3}}$ and $\frac{5\pi}{3}$

since $[0, \pi]$

C

#35) $r = \theta - 2\cos\theta$

we know that

$x = r\cos\theta$ and $y = r\sin\theta$

$2 = r\cos\theta$

$2 = (\theta - 2\cos\theta)\cos\theta$ → solve by calc

$y = 2$
$y = \cos\theta(\theta - 2\cos\theta)$
where they intersect on $[0, 2\pi]$
$[0, 6.28]$

$\theta = 5.201$

thus plug into

$y = r\sin\theta$

$y = (\theta - 2\cos\theta)\sin\theta$

$[5.201 - 2\cos(5.201)]\sin(5.201)$ by calc

$\boxed{y = -3.763}$ B