Homework Exercises

#1) $y^3 - xy^2 = 4$ at point $\boxed{y=2}$ $fg' + gf'$

$$3y^2 \frac{dy}{dx} - \left[x \cdot 2y \frac{dy}{dx} + y^2 \cdot 1\right] = 0$$

if $y=2$
$2^3 - x2^2 = 4$
$\cancel{8} - 4x = 4$
$\cancel{-8} \quad -8$
$-4x = -4$
$\boxed{x=1}$

$$3y^2 \frac{dy}{dx} - 2xy\frac{dy}{dx} - y^2 = 0$$

$$\frac{\cancel{(3y^2 - 2xy)}}{\cancel{3y^2 - 2xy}} \frac{dy}{dx} = \frac{y^2}{3y^2 - 2xy}$$

$$\frac{dy}{dx} = \frac{y^2}{3y^2 - 2xy}$$

plug in $x=1$, $y=2$

$$\frac{dy}{dx} = \frac{2^2}{3(2)^2 - 2(1)(2)} = \frac{4}{12-4} = \frac{4}{8} = \boxed{\frac{1}{2}}$$

D

$fg' + gf'$

#2) $y^2 - xy - 3x = 1$ at point $(0,-1)$

$2y\frac{dy}{dx} - \left[x \cdot 1 \frac{dy}{dx} + y \cdot 1\right] - 3 = 0$

$2y\frac{dy}{dx} - x\frac{dy}{dx} - y - 3 = 0$

$\frac{(2y-x)\frac{dy}{dx}}{2y-x} = \frac{y+3}{2y-x}$

$\frac{dy}{dx} = \frac{y+3}{2y-x}$

plug in point $(0,-1)$

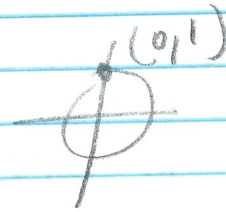
$\frac{dy}{dx} = \frac{-1+3}{2(-1)-0} = \frac{2}{-2} = \boxed{-1}$ A

$fg' + gf'$

#3) $y = x\sin x$ at point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

$y' = x \cdot \cos x \cdot 1 + \sin x \cdot 1$

$y' = x\cos x + \sin x$



plug in $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

$= \frac{\pi}{2}\cos\frac{\pi}{2} + \sin\frac{\pi}{2}$

$= \cancel{\frac{\pi}{2} \cdot 0} + 1$

$\boxed{m = 1}$

$y - y_1 = m(x - x_1)$

$y - \frac{\pi}{2} = 1(x - \pi/2)$

$y - \cancel{\frac{\pi}{2}} = x - \pi/2$

$\cancel{+\pi/2} \qquad + \pi/2$

$\boxed{y = x}$ E

$fg' + gf'$

#4) $y = xe^{-x}$ horizontal when $m=0$

$y' = x \cdot e^{-x} \cdot -1 + e^{-x} \cdot 1$

$y' = -xe^{-x} + e^{-x}$

so when $-xe^{-x} + e^{-x} = 0$

$e^{-x}(-x+1) = 0$

$\cancel{e^{-x} = 0}$ $-x+1=0$

$\boxed{x = 1}$ B

#5) $y = x^5 + x^3 - 2x = 0$ $x(x^4 + x^2 - 2)$

$x(x^2+2)(x^2-1)=0$

$(x+1)(x-1)$

$x=0$ $x=1,-1$

$y' = 5x^4 + 3x^2 - 2 = 0$ find critical points

let $w = x^2$

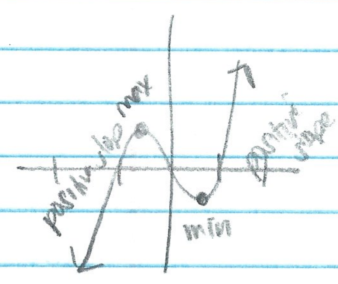
$5w^2 + 3w - 2 = 0$

$(w+1)(5w-2) = 0$

$w = -1$ $w = \frac{2}{5}$

$\cancel{x^2 = -1}$ $x^2 = \frac{2}{5}$

$x = \pm\sqrt{\frac{2}{5}}$



$y'' = 20x^3 + 6x = 0$

$2x(10x^2+3) = 0$

$2x = 0$ $\cancel{10x^2+3=0}$

$x = 0$ inflection point

slope at $x=0$

$y'' = 20x^3 + 6x$

$y''(\sqrt{\frac{2}{5}}) > 0$ concave up

local min $y'(0) = \boxed{-2}$

$m=$

#6) $x^2-y^2=12$ $(4,2)$

$2x-2y\frac{dy}{dx}=0$

$-2y\frac{dy}{dx}=-2x$

$\frac{dy}{dx}=\frac{x}{y}=\frac{4}{2}=2$ $\boxed{m=2}$

$y=mx+b$

$2=2(4)+b$

$2=8+b$

$-8\ \ -8$

$-6=b$

$\boxed{y=2x-6}$ C

$fg'+gf'$

#7) $y^2-xy+9=0$ vertical = undefined slope

$2y\frac{dy}{dx}-[x\cdot\frac{dy}{dx}+y\cdot 1]+0=0$

$2y\frac{dy}{dx}-x\frac{dy}{dx}-y=0$

$(2y-x)\frac{dy}{dx}=\frac{y}{2y-x}$

$\frac{dy}{dx}=\frac{y}{2y-x}$ undefined when the denominator $=0$

$2y-x=0$

$x=2y$

$y^2 - xy + 9 = 0$

$x = 2y$

$y^2 - 2y \cdot y + 9 = 0$

$y^2 - 2y^2 + 9 = 0$

$-y^2 = -9$

$y^2 = 9$

$\boxed{y = \pm 3}$ D

#9) $x = 3$ $2x^2 - y3 = 10$

$y = 2$

when $x = 3.04$

$2(3.04)^2 - y3 = 10$

$18.4832 - y3 = 10$

$-y^2 = -8.4832$

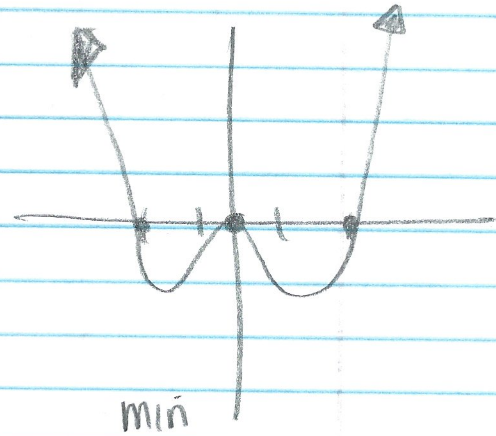
$\sqrt[3]{y^3} = \sqrt[3]{8.4832}$

$\boxed{y = 2.04}$ C

#12) $f(x) = x^4 - 4x^2$ zeros $x^2(x^2-4)$

$x=0$ $x=\pm 2$

$f'(x) = 4x^3 - 8x = 0$



$4x(x^2-2)=0$

$4x=0$ $x^2-2=0$

$x=0$ $x^2=\sqrt{2}$

$x=\pm\sqrt{2}$

max	min	min
$x=0$	$x=\sqrt{2}$	$x=-\sqrt{2}$
$\wedge$	$\wedge$	$\wedge$
$x=-1$ $x=1$	1 1.5	-1 1.5
$-4+8$ $4-8$		$-4+8$
$+$ $-$	$-$ $+$	$+$ $-$

$\boxed{\begin{matrix}2\text{ min}\\1\text{ max}\end{matrix}}$ E

#13) inflection points

$y'' = 12x^2 - 8 = 0$

$4(3x^2-2)=0$

$\frac{3x^2}{3} = \frac{2}{3}$

$\sqrt{x^2} = \sqrt{\frac{2}{3}}$

$x = \pm\sqrt{\frac{2}{3}}$ $\boxed{2\text{ points}}$ C

#14) $y=-4\sqrt{2-x} = -4(2-x)^{1/2}$

$y' = -\cancel{4}^{2}\cdot\frac{1}{\cancel{2}}(2-x)^{-1/2}\cdot -1$

$= \frac{2}{(2-x)^{1/2}} = \frac{2}{\sqrt{2-x}} = 0$

undefined when $x=2$

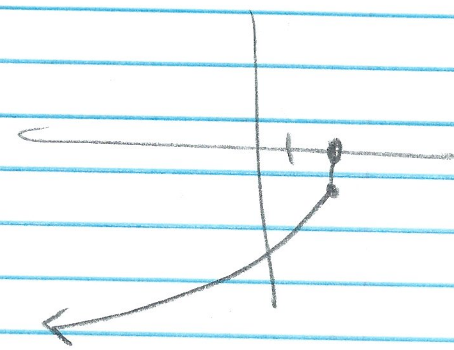
critical point at $x=2$

$X=2$

$X=1$ $\quad$ ~~$X=3$~~ not possible

$+$

increasing to $x=2$



make value find y coordinate

$y=-4(2-2)^{1/2}$

$y=0$ $\quad$ $(2,0)$ $\quad$ $\boxed{0}$ $\quad$ A

#5) $f'(x) = x(x-3)^2(x+1)^4 = 0$

$x=0 \quad x=3 \quad x=-1$

$x=-1 \qquad x=0 \qquad x=3$

$x=-2 \quad x=-.5 \quad x=1 \quad x=4$

$- \quad - \quad + \quad +$

min

only min is at $x=0$ $\boxed{I}$ B

#10) f' negative slope f'' negative
concave down

$\boxed{E}$

#17) f'' positive
concave up

f' negative
negative slope

$\boxed{B}$

#18–21: $s = t^3 - 6t^2 + 12t - 8$

#18) $s' = 3t^2 - 12t + 12 = 0$ $\qquad$ $s'' = 6t - 12$

$3(t^2 - 4t + 4)$

4

-2 $\quad$ -2

-4

$3(t-2)(t-2) = 0$

$t = 2 \rightarrow$ slope $= 0$

$t = 1$ $\qquad$ $t = 3$

$3 - \cancel{12 + 12}$ $\qquad$ $27 - 36 + 12$

$+$ $\qquad$ $+$ $\quad$ Increasing everywhere else

$\boxed{\text{so increasing on all } t \text{ except } t = 2}$ B

#19) speed $= \frac{dy}{dx} = y' = 3t^2 - 12t + 12$

slope is increasing on all t, but at $x = 2$
so positive
the slope is 0

Thus slope $= \boxed{0}$ is the minimum speed $\quad$ D

#20) Acceleration $y'' = 6t - 12 = 0$

$6t = 12$

$t = 2$

$t = 1$ $\quad$ $t = 3$

$-$ $\quad$ $+$

$t < 2$
negative
acceleration

$t > 2$
positive
acceleration

and velocity
is positive

A

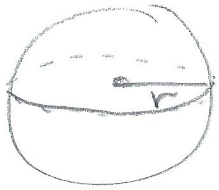
#21) if a and c are opposite sign, then
acceleration is decreasing

for $t < 2$
acceleration is negative
velocity is positive

thus, the acceleration is decreasing on
$t < 2$

Exercises

#31) balloon is being filled = volume



$$\frac{dV}{dt} = 4$$

volume of a sphere is $V = \frac{4}{3}\pi r^3$

surface area of a sphere is $S = 4\pi r^2$

$$\frac{dV}{dt} = 4$$

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}r^2 \cdot \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$\frac{4}{4} = \frac{4\pi r^2}{4}\frac{dr}{dt}$$

$$\frac{1}{\pi r^2} = \frac{\pi r^2}{\pi r^2}\frac{dr}{dt}$$

$$\frac{1}{\pi r^2} = \frac{dr}{dt}$$

$$\frac{1}{\pi \cdot 2^2} = \frac{dr}{dt}$$

$$\boxed{\frac{1}{4\pi} = \frac{dr}{dt}}$$

Find r

$$V = \frac{32\pi}{3}$$

$$\frac{3}{4} \cdot \frac{\overset{8}{\cancel{32}}\pi}{\cancel{3}} = \frac{\cancel{4}}{\cancel{3}}\pi r^3 \cdot \frac{3}{4}$$

$$\frac{8\cancel{\pi}}{\cancel{\pi}} = \frac{\cancel{\pi} r^3}{\cancel{\pi}}$$

$$8 = r^3$$

$$\boxed{r = 2}$$

$$\frac{dS}{dt} = 4\pi \cdot 2r\frac{dr}{dt}$$

$$\frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt}$$

*Need to find r =

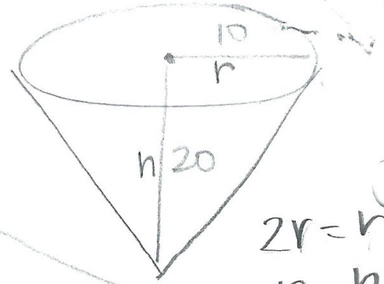
and $\frac{dr}{dt}$ =

$$\frac{dS}{dt} = 8\pi \cdot (2)\left(\frac{1}{4\pi}\right)$$

$$\frac{dS}{dt} = \frac{16\cancel{\pi}}{4\cancel{\pi}}$$

$$\frac{dS}{dt} = \boxed{4}$$ C

#32) reservior = volume



Volume of a cone $\Rightarrow$ $V = \frac{1}{3}\pi r^2 h$

$2r = h$

$r = \frac{h}{2}$

surface is falling at a rate of $\frac{1}{2}$

$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$\frac{dh}{dt} = -\frac{1}{2}$ when $h = 8$

$V = \frac{1}{3}\pi \frac{h^2}{4} \cdot h$

$V = \frac{1}{12}\pi h^3$

Find $\frac{dV}{dt} = \frac{1}{12}\pi \cdot 3h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

need to find

$h = 8$

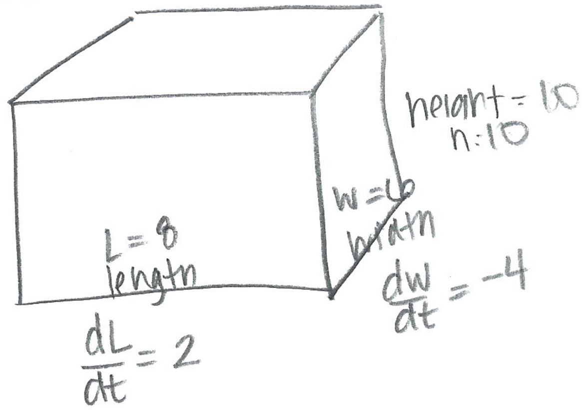
$-\frac{1}{2}$

$\frac{dV}{dt} = \frac{1}{4}\pi(8)^2\left(-\frac{1}{2}\right)$

$\frac{dV}{dt} = \frac{-64\pi}{8} = -8\pi$

rate $= \boxed{8\pi}$

#42)



$V = L \times w \times h$

$V = L \times w \times 10$

$V = 10(L \times w)$ $fg' + gf'$

Find $\frac{dV}{dt}$

$$\frac{dV}{dt} = 10\left(L \cdot \frac{dw}{dt} + w \cdot \frac{dL}{dt}\right)$$

$$= 10(8 \cdot (-4) + 6 \cdot 2)$$

$$= 10(-32 + 12)$$

$$= 10(-20)$$

$$= \boxed{-200}$$

D