Practice Exercises

Barons MC
pg#104: 1-31 odd

1) $\lim_{x\to 2} \frac{x^2-4}{x^2+4} = \frac{2^2-4}{2^2+4} = \frac{4-4}{4+4} = \frac{0}{8} = \boxed{0}$ A

#2) $\lim_{x\to\infty} \frac{4-x^2}{x^2-1} = \frac{\frac{4}{x^2} - \frac{x^2}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}} = \lim_{x\to\infty} \frac{\cancel{\frac{4}{x^2}}^{\to 0} - 1}{1 - \cancel{\frac{1}{x^2}}^{\to 0}} = \boxed{-1}$ D

#3) $\lim_{x\to 3} \frac{x-3}{x^2-2x-3} = \lim_{x\to 3} \frac{\cancel{(x-3)}}{\cancel{(x-3)}(x+1)} = \lim_{x\to 3} \frac{1}{x+1} = \frac{1}{3+1}$

$= \boxed{\frac{1}{4}}$ C

#4) $\lim_{x\to 0} \frac{x}{x} = \lim_{x\to 0} 1 = \boxed{1}$ A

#5) $\lim_{x\to 2} \frac{x^3-8}{x^2-4} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{(x+2)\cancel{(x-2)}}$

$= \lim_{x\to 2} \frac{x^2+2x+4}{x+2} = \frac{2^2+2(2)+4}{2+2}$

$= \frac{4+4+4}{4} = \frac{12}{4} = \boxed{3}$

D

#6) $\lim_{x\to\infty} \dfrac{4-x^2}{4x^2-x-2} = \lim_{x\to\infty} \dfrac{\frac{4}{x^2}-\frac{x^2}{x^2}}{\frac{4x^2}{x^2}-\frac{x}{x^2}-\frac{2}{x^2}}$

$= \lim_{x\to\infty} \dfrac{\frac{4}{x^2}^{\to 0}-1}{4-\frac{1}{x}^{\to 0}-\frac{2}{x^2}^{\to 0}} = \boxed{-\frac{1}{4}}$

#7) $\lim_{x\to-\infty} \dfrac{5x^3+27}{20x^2+10x+9}$ higher power on top

$\boxed{-\infty}$ A

#8) $\lim_{x\to\infty} \dfrac{3x^2+27}{x^3-27} = \boxed{0}$ E

higher power on bottom

#9) $\lim_{x\to\infty} \dfrac{2^{-x}}{2^x} = \lim_{x\to\infty} \dfrac{1}{2^x \cdot 2^x} = \lim_{x\to\infty} \dfrac{1}{2^{2x}} = \frac{1}{\infty} = \boxed{0}$

C

#10) $\lim_{x\to-\infty} \dfrac{2^{-x}}{2^x} = \lim_{x\to\infty} \dfrac{1}{2^{2x}} = \boxed{\infty}$ D

#11) $\lim_{x\to 0} \dfrac{\sin 5x}{x} \cdot \dfrac{5}{5} = \lim_{x\to 0} 5\dfrac{\sin 5x}{5x} = 5 \lim_{x\to 0} \dfrac{\sin 5x}{5x}$

$= 5 \cdot 1 = \boxed{5}$

D

#12) $\lim\limits_{x\to 0} \frac{\sin 2x}{3x} \cdot \frac{2}{2} = \lim\limits_{x\to 0} \frac{2\sin 2x}{3 \cdot 2x}$

$= \frac{2}{3} \lim\limits_{x\to 0} \frac{\sin 2x}{2x}$

$= \frac{2}{3} \cdot 1 = \boxed{\frac{2}{3}}$ B

#13) $y = \tan^{-1} x$ graph

Since the graph of $y = \tan x$ has vertical asymptotes at $x = \pm \pi/2$

Then the graph $y = \tan^{-1} x$ have horizontal asymptotes at $y = \pm \pi/2$ $\boxed{B}$

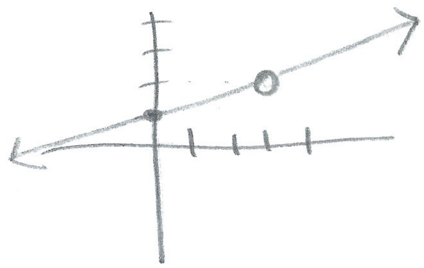
hole at $x=3$

#14) $y = \frac{x^2-9}{3x-9}$ we know that $\frac{x^2-9}{3x-9} = \frac{(x+3)\cancel{(x-3)}}{3\cancel{(x-3)}} = \frac{x+3}{3}$ line $\frac{1}{3}x+1$

However

$f(3) = \frac{3+3}{3} = \frac{6}{3} = 2.$

pg# 94-95



~~horizontal asymptote~~

$\lim\limits_{x\to\infty} y =$

$\lim\limits_{x\to\infty} \frac{x^2-9}{3x-9} = \infty$

~~vertical asymp.~~

$\lim\limits_{x\to a^-} y = \pm\infty$

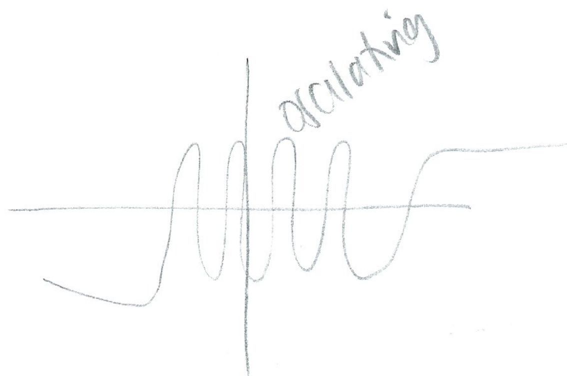
$\lim\limits_{x\to 3^-} \frac{x^2-9}{3x-9} = \lim\limits_{x\to 3^-} \frac{x+3}{3} \neq \infty / -\infty$

#15) $\lim\limits_{x\to 0} \frac{\sin x}{x^2+3x} = \lim\limits_{x\to 0} \frac{\sin x}{x(x+3)} = \lim\limits_{x\to 0} \frac{\sin x}{x} \cdot \lim\limits_{x\to 0} \frac{1}{x+3}$

$= 1 \cdot \frac{1}{0+3}$

$= 1 \cdot \frac{1}{3} = \boxed{\frac{1}{3}}$ B

#16) $\lim\limits_{x\to 0} \sin\frac{1}{x}$



since it is osilating near zero the limit does not exist $\boxed{C}$

#17) $y = \frac{2x^2+4}{2+7x-4x^2}$

horizontal $y = \frac{2}{-4}$

$y = -\frac{1}{2}$

vertical asymptote

$-\frac{8}{4} + \frac{8}{4} = 0$

$-4x^2+7x+2$ $\quad -4 \cdot \frac{1}{16} = -\frac{1}{4} + -\frac{7}{4} + 2$

$\frac{-2x^2-4}{4x^2-7x-2}$

$-1(4x-7x-2)$

-8

$x \quad 4x \quad 4x$

$-2 \quad 8 \quad 1$

-7

$-(x-2)(4x+1)=0$

$x=2$ $\quad \boxed{x=-\frac{1}{4}}$ A

#18) $\lim_{x \to \infty} \frac{2x^2-1}{(2-x)(2+x)} = \lim_{x \to \infty} \frac{2x^2-1}{4-x^2} = \frac{2}{-1}$

$= \boxed{-2}$ B

since the exponents are the same top and bottom use coefficents

#19) $\lim_{x \to 0} \frac{|x|}{x}$ we know that

$|x| = \begin{cases} x & \text{when } x > 0 \\ -x & \text{when } x < 0 \end{cases}$

So if we look at the left and right hand limits

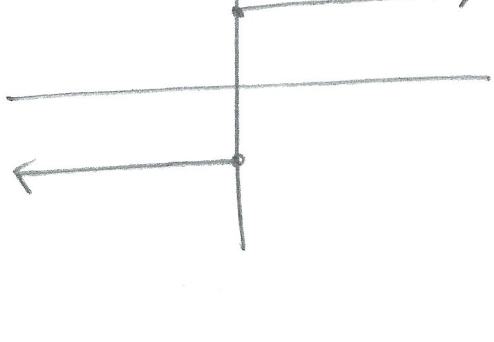
$\lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} \frac{x}{x} = \lim_{x \to 0^+} 1 = 1$

but

$\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} \frac{-x}{x} = \lim_{x \to 0^-} -1 = -1$

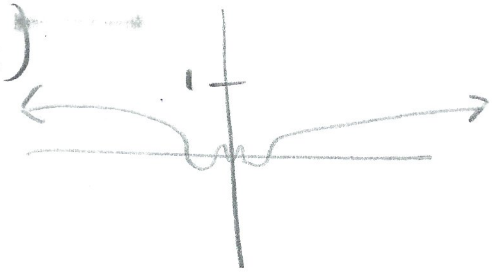
the L/R hand limits are different

so the limit does not exist $\boxed{B}$



#20) $\lim\limits_{x\to\infty} x\sin\frac{1}{x} = \lim\limits_{x\to\infty} x \cdot \lim$

graph of $x\sin(\frac{1}{x})$



$\lim\limits_{x\to\infty} \dfrac{\sin(\frac{1}{x})}{\frac{1}{x}}$

#21) $\lim\limits_{x\to\infty} \dfrac{\cos(\frac{1}{x})\cdot -\frac{1}{x^2}}{-\frac{1}{x^2}} = \cos(\frac{1}{\infty}) = \cos(0) = \boxed{1}$

#21) $\lim\limits_{x\to\pi} \dfrac{\sin(\pi-x)}{\pi-x}$ ~~we know that~~ $\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$

$\dfrac{\sin(0)}{0} = \dfrac{0}{0}$

~~as $x\to\pi$ $(\pi-x)\to 0$~~

~~so $\lim\limits_{x\to\pi} \dfrac{\sin(\pi-x)}{\pi-x} = \boxed{1}$~~

L'Hopitals

$\lim\limits_{x\to\pi} \dfrac{\cos(\pi-x)\cdot -1}{-1} =$

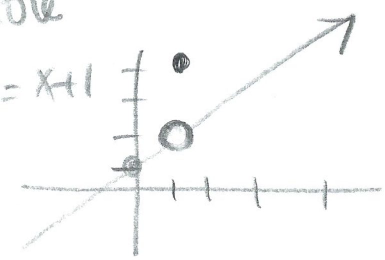
$\dfrac{\cos(0)\cdot \cancel{-1}}{\cancel{-1}} = \boxed{1}$

#22) $f(x) = \begin{cases} \dfrac{x^2-1}{x-1} & \text{if } x\neq 1 \\ 4 & \text{if } x=1 \end{cases}$

$\lim\limits_{x\to 1} f(x)$ exists ✓

$f(1) = 4$ exists ✓

$\dfrac{x^2-1}{x-1} = \dfrac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = x+1$ hole



f is not continuous at $x=1$ since there is a hole at $x=1$

$\boxed{C}$ I and II

23)

$$\begin{cases} f(x) = \dfrac{x^2 - x}{2x} & \text{for } x \neq 0 \\ f(0) = k \end{cases}$$

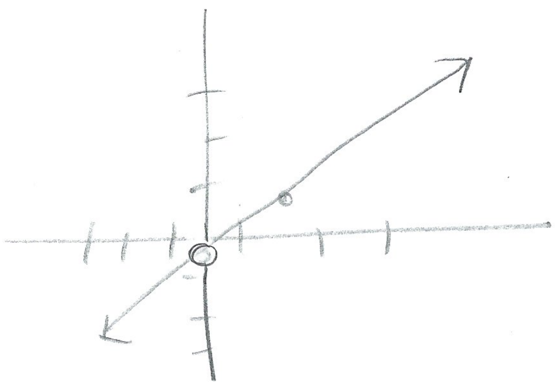
hole at $x=0$

$$f(x) = \frac{\cancel{x}(x-1)}{2\cancel{x}} = \frac{x-1}{2}$$

plug in $x=0$

$$\frac{0-1}{2} = \frac{-1}{2}$$

$$= \frac{1}{2}x - \frac{1}{2}$$



since our hole is at $x=0$

when $x=0$
$y = \frac{-1}{2}$

so for f to be continuous

$f(0) = \frac{-1}{2}$

so $\boxed{k = \frac{-1}{2}}$

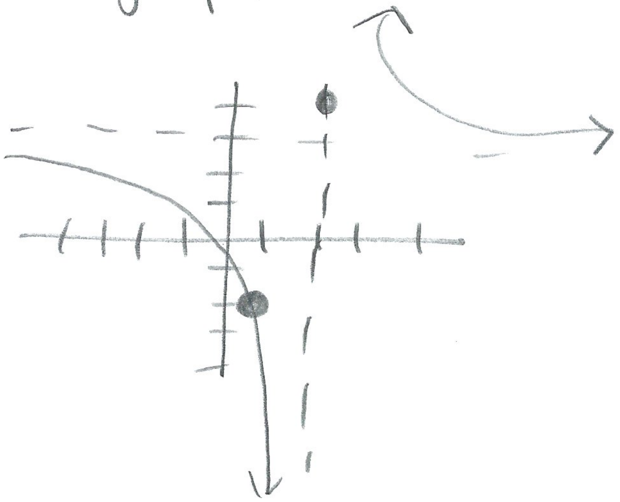
#24) $\begin{cases} f(x) = \dfrac{3x(x-1)}{x^2-3x+2} & x \neq 1, 2 \\ f(1) = -3 \\ f(2) = 4 \end{cases}$

horizontal asym at $y = 3$

hole at $x = 1$

$$f(x) = \frac{3x(x-1)}{x^2-3x+2} = \frac{3x\cancel{(x-1)}}{(x-2)\cancel{(x-1)}} = \frac{3x}{x-2}$$

so we have a hole at $x = 1$
and a vertical asymptote at $x = 2$



using $\dfrac{3x}{x-2}$ plugin $x = 1$

$$\frac{3(1)}{1-2} = \frac{3}{-1} = -3$$

but above
$f(1) = -3$

so f is continuous at $x = 1$, but not continuous at $x = 2$ since there is a vertical asymptote at $x = 2$

$\boxed{B}$

#25) $f(x) = \frac{4}{x^2-1} = \frac{4}{(x-1)(x+1)}$

vertical asymptote at $x=1$ and $x=-1$

horizontal asymptote $y = \frac{0x^2}{1x^2} = y = 0$
or the x-axis

$\boxed{C}$

#26) $y = \frac{2x^2+2x+3}{4x^2-4x} = \frac{2x^2+2x+3}{4x(x-1)}$

vertical asymptotes at
$\boxed{x=0 \text{ and } x=1}$

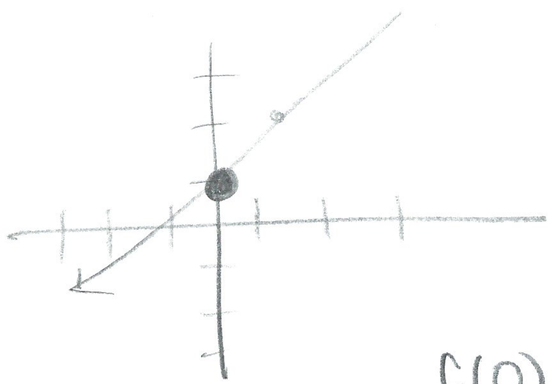
horizontal asymptote at
$y = \frac{2}{4}$ $\boxed{y = \frac{1}{2}}$

$\boxed{C}$

#27) let $f(x) = \begin{cases} \dfrac{x^2+x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

$$\frac{x^2+x}{x} = \frac{x(x+1)}{x} = x+1$$

hole at $x=0$



but when $x=0$
$y=1$

$f(0)=1$ so it exists ✓

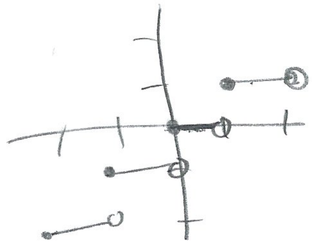
the $\lim_{x\to 0} f(x) =$ exists ✓.

f is continuous at $x=0$ ✓

$\boxed{D}$ (all)

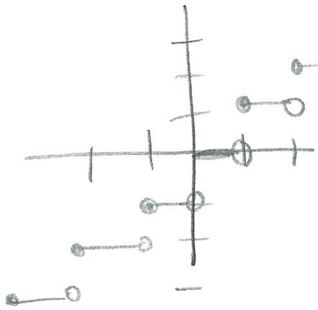
#28) greatest integer function pg# 90

$$\lim_{x\to \frac{1}{2}} [x] = 0$$



$\boxed{D}$

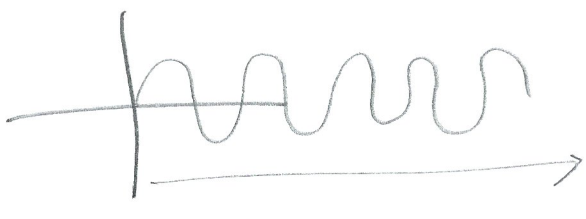
#29) $\lim_{x\to 2} [x]$



$\lim_{x\to 2^-} [x] = +3$ $\lim_{x\to 2^+} [x] = -2$

Since the left and right limits are different the limit $\boxed{\text{DNE}}$ E

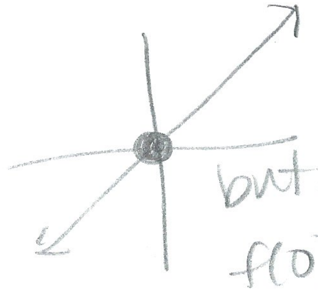
#30) $\lim_{x\to \infty} \sin x$



Since the limits oscillates between -1 and 1 the limit DNE $\boxed{E}$

#31) $f(x) = \begin{cases} \frac{x^2}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ $\quad \frac{x^2}{x} = x$

hole at $x = 0$



but since $f(0) = 0$

$\boxed{A}$ continuous everywhere

#32) $\lim_{x \to 2} f(x) = \boxed{0}$ A

#33) $[-1, 3]$ $\boxed{E}$ defined at every point

#34) removable discontinuity at $x = 2$

$\boxed{C}$

#35) continuous $0 < x < 1$ $\boxed{B}$

#36) jump discontinuity $\boxed{x = 1}$

$\boxed{B}$