Name, Answer key (1)

Calculus BC: Section I

Section I consists of 45 multiple-choice questions. Part A contains 28 questions and does not allow the use of a calculator. Part B contains 17 questions and requires a graphing calculator for some questions. Twenty-four sample multiple-choice questions for Calculus BC are included in the following sections. Answers to the sample questions are given on page 39.

Part A Sample Multiple-Choice Questions

A calculator may not be used on this part of the exam.

Part A consists of 28 questions. Following are the directions for Section I, Part A, and a representative set of 14 questions.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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1. A curve is described by the parametric equations $x = t^2 + 2t$ and $y = t^3 + t^2$. An equation of the line tangent to the curve at the point determined by $t = 1$ is

   (A) $2x - 3y = 0$
   (B) $4x - 5y = 2$
   (C) $4x - y = 10$
   (D) $5x - 4y = 7$
   (E) $5x - y = 13$

$t=1$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2+2t}{2t+2} = \frac{3(1)^2+2(1)}{2(1)+2} = \frac{5}{4}$$

At $t=1$

$x = 1^2+2(1) = 3 \quad y = 1^3+1^2 = 2$

$(3,2)$

$y = mx+b$

$y = \frac{5}{4}x+b$

$2 = \frac{5}{4}(3)+b$

$\frac{8}{4} \quad 2 = \frac{15}{4}+b$

$-\frac{15}{4} \quad -\frac{15}{4}$

$-7/4 = b$

$\left(y = \frac{5}{4}x - \frac{7}{4}\right)4$

$4y = 5x-7$

$\boxed{5x-4y=7}$ D

$fg' + gf'$

2. If $3x^2 + 2xy + y^2 = 1$, then $\frac{dy}{dx} =$

   (A) $-\frac{3x+y}{y^2}$

   (B) $-\frac{3x+y}{x+y}$

   (C) $\frac{1-3x-y}{x+y}$

   (D) $-\frac{3x}{1+y}$

   (E) $-\frac{3x}{x+y}$

$6x \cdot dx + 2[x \cdot 1 dy + y \cdot 1 dx] + 2y\,dy = 0$

$6x\,dx + 2x\,dy + 2y\,dx + 2y\,dy = 0$

$(2x+2y)dy = (-6x-2y)dx$

$\frac{dy}{dx} = \frac{-6x-2y}{2x+2y} = \frac{-3x-y}{x+y}$

$\boxed{-\frac{3x+y}{x+y}}$ B

x	$g'(x)$	y $g(x)$
−1.0	2	−2 y_0
−0.5	4	
0.0	3	
0.5	1	1 y_1
1.0	0	
1.5	−3	
2.0	−6	2.5 y_2

$h = 1.5$ jump

jump 1.5

since $g(-1) = -2$

$y_1 = y_0 + h\,g'(x_0)$

$y_1 = -2 + (1.5)2$

$= 1$

$y_2 = y_1 + h(g'(x_1))$

$= 1 + (1.5)(1)$

$= 2.5$

3. The table above gives selected values for the derivative of a function g on the interval $-1 \le x \le 2$. If $g(-1) = -2$ and Euler's method with a step-size of 1.5 is used to approximate $g(2)$, what is the resulting approximation?

   (A) −6.5
   (B) −1.5
   (C) 1.5
   (D) 2.5
   (E) 3

4. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{n3^n}{x^n}$ converges?

   (A) All x except $x = 0$
   (B) $|x| = 3$
   (C) $-3 \le x \le 3$
   (D) $|x| > 3$
   (E) The series diverges for all x.

$-1 < \frac{x}{3} < 1$

$-1 > \frac{x}{3} > 1$

$-3 > x > 3$

$|x| > 3$

$\lim_{n\to\infty} \left| \frac{(n+1)3^{n+1}}{x^{n+1}} \cdot \frac{x^n}{n3^n} \right| = \lim_{n\to\infty} \left| \frac{(n+1)\cancel{3^n} \cdot 3}{\cancel{x^n} \cdot x} \cdot \frac{\cancel{x^n}}{n\cancel{3^n}} \right|$

$\lim_{n\to\infty} \frac{3n+3}{n} \cdot \frac{1}{x}$

$\frac{3}{|x|} < 1$ $\boxed{|x| > 3}$

$x > 3$ $\frac{3}{x} < 1$ $\frac{3}{x} > -1$ $x < 3$ $x > -3$

$f'(x)=g(x)$

5. If $\frac{d}{dx}f(x) = g(x)$ and if $h(x) = x^2$, then $\frac{d}{dx}f(h(x)) =$ $f'(h(x))\cdot h'(x)$

   $h'(x)=2x$

   $g(x^2)\cdot 2x$

   $2xg(x^2)$

   (A) $g(x^2)$
   (B) $2xg(x)$
   (C) $g'(x)$
   (D) $2xg(x^2)$
   (E) $x^2g(x^2)$

6. If F' is a continuous function for all real x, then $\lim\limits_{h\to 0}\frac{1}{h}\int_a^{a+h} F'(x)\,dx$ is

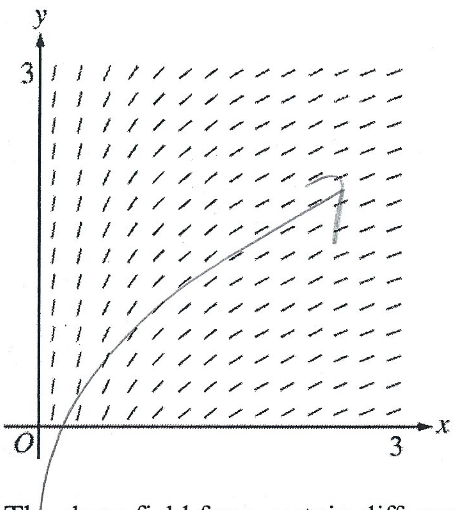
   (A) 0
   (B) ~~$F(0)$~~
   (C) ~~$F(a)$~~
   (D) ~~$F'(0)$~~
   (E) $F'(a)$

   $\lim\limits_{h\to 0}\frac{f(a+h)-f(a)}{h}$

   Definition of derivative

   $F'(a)$

   $\lim\limits_{h\to 0}\frac{1}{h}\left[F(a+h)-F(a)\right] = \lim\limits_{h\to 0}\frac{F(a+h)-F(a)}{h}$



7. The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

   (A) $y = x^2$
   (B) $y = e^x$
   (C) $y = e^{-x}$
   (D) $y = \cos x$
   (E) $y = \ln x$

$\lim_{b\to 1^-}\int_0^b \frac{1}{(1-x)^2} + \lim_{b\to 1^+}\int_b^3 \frac{1}{(1-x)^2}$

$\frac{1}{(1-b)} - \left(\frac{1}{1-0}\right)$

$(1-x)^{-2}$ $\infty -$

$\frac{-(1-x)^{-1}}{-1}$

$\frac{1}{(1-x)}$

8. $\displaystyle\int_0^3 \frac{dx}{(1-x)^2}$ is

(A) $-\frac{3}{2}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) $\frac{3}{2}$

(E) divergent

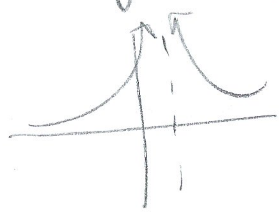
$u = 1-x$

$du = -1\,dx$

$-du = -dx$

$\cancel{\int_0^3 \frac{1}{u^2}\,du}$ u^{-2}

horizontal asym at $x=1$



are under the curve $= \infty$

9. Which of the following series converge to 2?

I. $\displaystyle\cancel{\sum_{n=1}^{\infty} \frac{2n}{n+3}}$ $\lim_{n\to\infty} \frac{2n}{2n+3} = 2$ Thus nth series test $\neq 0$ Diverges

II. $\displaystyle\sum_{n=1}^{\infty} \frac{-8}{(-3)^n}$ $-8\left(-\frac{1}{3}\right)^n$ $\frac{a}{1-r} = \frac{\frac{8}{3}}{1+\frac{1}{3}} = \frac{\frac{8}{3}}{\frac{4}{3}} = \frac{8}{3}\cdot\frac{3}{4} = 2$

III. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^n}$ $\left(\frac{1}{2}\right)^n = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 1\cdot\frac{2}{1} = 2$

(A) I only
(B) II only
(C) III only
(D) ~~I and III only~~
(E) II and III only

10. If the function f given by $f(x) = x^3$ has an average value of 9 on the closed interval $[0, k]$, then $k =$

(A) 3

(B) $3^{1/2}$

(C) $18^{1/3}$

(D) $36^{1/4}$

(E) $36^{1/3}$

$9 = \frac{1}{k-0}\int_0^k x^3$

$\frac{1}{k}\frac{x^4}{4}\Big|_0^k$

$\frac{1}{k}\cdot\frac{k^4}{4} - \cancel{\frac{1}{k}\cdot\frac{0^4}{4}}$

$\frac{k^3}{4} = 9$ $\sqrt[3]{k^3} = \sqrt[3]{36}$

$k = 36^{1/3}$

11. Which of the following integrals gives the length of the graph $y = \sin(\sqrt{x})$ between $x = a$ and $x = b$, where $0 < a < b$?

arc length $\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

(A) $\int_a^b \sqrt{x + \cos^2(\sqrt{x})}\, dx$

(B) $\int_a^b \sqrt{1 + \cos^2(\sqrt{x})}\, dx$

(C) $\int_a^b \sqrt{\sin^2(\sqrt{x}) + \frac{1}{4x}\cos^2(\sqrt{x})}\, dx$

(D) $\int_a^b \sqrt{1 + \frac{1}{4x}\cos^2(\sqrt{x})}\, dx$

(E) $\int_a^b \sqrt{\frac{1 + \cos^2(\sqrt{x})}{4x}}\, dx$

$\int_a^b \sqrt{1+\left(\frac{\cos(\sqrt{x})}{2\sqrt{x}}\right)^2}$ $\quad \frac{dy}{dx} = \cos(\sqrt{x})\cdot\frac{1}{2}x^{-1/2}$

$\int_a^b \sqrt{1+\frac{\cos^2(\sqrt{x})}{4x}}\,dx \quad \frac{dy}{dx} = \frac{\cos\sqrt{x}}{2\sqrt{x}}$

12. Which of the following integrals represents the area enclosed by the smaller loop of the graph of $r = 1 + 2\sin\theta$?

(A) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1 + 2\sin\theta)^2\, d\theta$

(B) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1 + 2\sin\theta)\, d\theta$

(C) $\frac{1}{2}\int_{-\pi/6}^{7\pi/6} (1 + 2\sin\theta)^2\, d\theta$

(D) $\int_{-\pi/6}^{7\pi/6} (1 + 2\sin\theta)^2\, d\theta$

(E) $\int_{7\pi/6}^{-\pi/6} (1 + 2\sin\theta)\, d\theta$

$1+2\sin\theta = 0$

$2\sin\theta = -1$

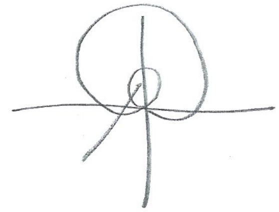
$\sin\theta = -\frac{1}{2}$

$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)\ \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\frac{7\pi}{6} \quad \frac{11\pi}{6}$

$\int_{7\pi/6}^{11\pi/6}$

$\text{Area} = \frac{1}{2}\int (r(\theta))^2\, d\theta$



$\text{Area} = \boxed{\frac{1}{2}\int_{7\pi/6}^{11\pi/6} (1+2\sin\theta)^2\, d\theta}$

πr^2

$(1+2\sin\theta)^2$

13. The third-degree Taylor polynomial about $x = 0$ of $\ln(1 - x)$ is

(A) $-x - \frac{x^2}{2} - \frac{x^3}{3}$

(B) $1 - x + \frac{x^2}{2}$

(C) $x - \frac{x^2}{2} + \frac{x^3}{3}$

(D) $-1 + x - \frac{x^2}{2}$

(E) $-x + \frac{x^2}{2} - \frac{x^3}{3}$

$f(0) = \ln(1-0) = \ln(1) = 0$ $\qquad -1(1-x)^{-1}$

$\frac{f'(0)x}{1!}$ $\quad f'(x) = \frac{1}{1-x} \cdot -1 = \frac{-1}{1-x} = -1 \Rightarrow -x$

$\frac{f''(0)x^2}{2!}$ $\quad f''(x) = 1(1-x)^{-2} \cdot -1 = \frac{-1}{(1-x)^2} = \frac{-1}{1} =$

$\frac{-1x^2}{2!}$

14. If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

$(0,5)$

(A) $e^{\tan x} + 4$

(B) $e^{\tan x} + 5$

(C) $5e^{\tan x}$

(D) $\tan x + 5$

(E) $\tan x + 5e^x$

$\int \frac{dy}{y} = \int \sec^2 x \, dx$

$\ln y = \tan x + c$

$\ln 5 = \tan 0 + c$

$c = \ln 5$

$e^{\ln y} = e^{\tan x + \ln 5}$

$y = e^{\tan x + \ln 5}$

$= e^{\tan x} \cdot e^{\ln 5}$

$= e^{\tan x} \cdot 5$

$\boxed{y = 5e^{\tan x}}$

Part B Sample Multiple-Choice Questions

A graphing calculator is required for some questions on this part of the exam.

Part B consists of 17 questions. Following are the directions for Section I, Part B, and a representative set of 10 questions.

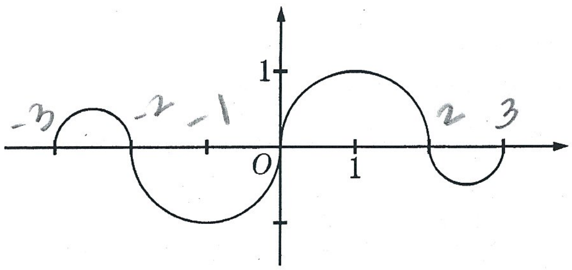
Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



Graph of f

15. The graph of the function f above consists of four semicircles. If $g(x) = \int_0^x f(t)\,dt$, where is $g(x)$ nonnegative? positive or zero

$\int_0^3 + \int_0^{-3} = \int_0^3 + -\int_{-3}^0 =$

(A) $[-3, 3]$
~~(B) $[-3, -2]$~~ $\cup [0, 2]$ only
(C) $[0, 3]$ only
(D) $[0, 2]$ only
(E) $[-3, -2] \cup [0, 3]$ only

16. If f is differentiable at $x = a$, which of the following could be false?

(A) f is continuous at $x = a$.
(B) $\lim\limits_{x \to a} f(x)$ exists.
(C) $\lim\limits_{x \to a} \dfrac{f(x) - f(a)}{x - a}$ exists. derivative
(D) $f'(a)$ is defined.
(E) $f''(a)$ is defined.

$g(3) = \int_0^3 f(t) = \pi - \frac{1}{4}\pi = \frac{3}{4}\pi$

$g(1) = \int_0^1 f(t) = \frac{\pi}{2}$

$g(-2)\ \int_0^{-2} = -\int_{-2}^0 =$

A) $\int_0^3 f(x) + -\int_{-3}^0 f(x)\,dx$ positive

$+ \quad - \quad + \quad =$

B)

C) $\int_0^3$

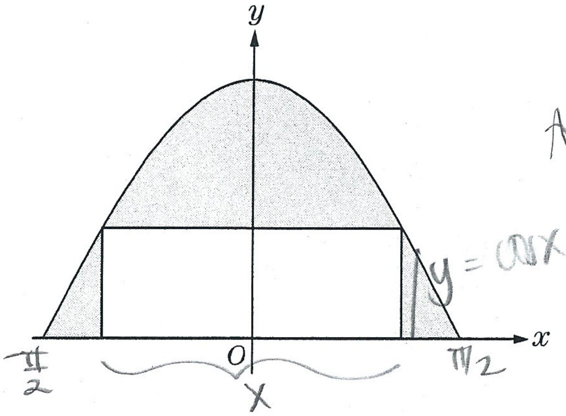
area under cos x

$\int_{-\pi/2}^{\pi/2} \cos x = 2$

Area of rectangle $= L \cdot W = 2x \cdot y$

$A = 2x\cos x$

area $= 2$



17. A rectangle with one side on the x-axis has its upper vertices on the graph of $y = \cos x$, as shown in the figure above. What is the minimum area of the shaded region?

$fg' + gf'$

Find max area of rectangle $A = 2x\cos x$

take derivative $= 0$

(A) 0.799
(B) 0.878
(C) 1.140
(D) 1.439
(E) 2.000

$A' = 2x \cdot -\sin x + \cos x \cdot 2$

$= -2x\sin x + 2\cos x = 0$ on calc

$= 2(\cos x - x\sin x) = 0$ $\quad \cos x = x\sin x$

$x = \frac{\cos x}{\sin x}$

$x = \cot x$

18. A solid has a rectangular base that lies in the first quadrant and is bounded by the x- and y-axes and the lines $x = 2$ and $y = 1$. The height of the solid above the point (x, y) is $1 + 3x$. Which of the following is a Riemann sum approximation for the volume of the solid?

(A) $\sum_{i=1}^{n} \frac{1}{n}\left(1 + \frac{3i}{n}\right)$

(B) $2\sum_{i=1}^{n} \frac{1}{n}\left(1 + \frac{3i}{n}\right)$

(C) $2\sum_{i=1}^{n} \frac{i}{n}\left(1 + \frac{3i}{n}\right)$

(D) $\sum_{i=1}^{n} \frac{2}{n}\left(1 + \frac{6i}{n}\right)$

(E) $\sum_{i=1}^{n} \frac{2i}{n}\left(1 + \frac{6i}{n}\right)$

volume $= x \cdot y \cdot f(x)$

$\sum_{i=1}^{n}$

= changing since from $\frac{2}{n}$ # of strip $\cdot 1 \cdot$

$x = .860$

$A = 2(.860)\cos(.860)$

$2 - (1.122)$

$.878$

$\boxed{\sum_{i=1}^{n} \frac{2}{n}\left(1 + 3\left(\frac{2i}{n}\right)\right)}$

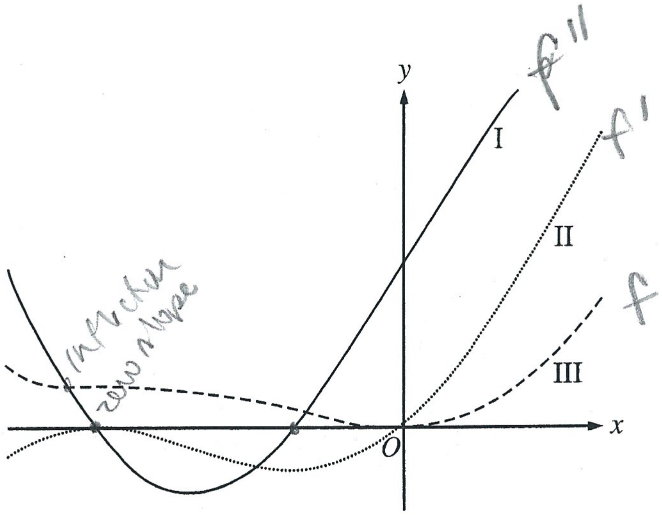
i^{th} slice

$\sum_{i=1}^{n} \frac{2}{n}\left(1 + \frac{6i}{n}\right.$

$h = 1 + 3x$

2

$f(x_i) = f(i \ast \Delta x) = f\left(i \times \frac{2}{n}\right) = f\left(\frac{2i}{n}\right) = 1 + 3\left(\frac{2i}{n}\right)$



19. Three graphs labeled I, II, and III are shown above. One is the graph of f, one is the graph of f', and one is the graph of f''. Which of the following correctly identifies each of the three graphs?

	f	f'	f''
(A)	I	II	III
(B)	I	III	II
(C)	II	I	III
(D)	II	III	I
(E)	III	II	I

20. A particle moves along the x-axis so that at any time $t \geq 0$ its velocity is given by $v(t) = \ln(t+1) - 2t + 1$. The total distance traveled by the particle from $t = 0$ to $t = 2$ is

(A) 0.667
(B) ~~0.704~~
(C) 1.540
(D) 2.667
(E) 2.901

$\int_0^{.79} v(t) - \int_{.79}^{2} v(t)$

$\boxed{1.54}$

21. If the function f is defined by $f(x) = \sqrt{x^3 + 2}$ and g is an antiderivative of f such that $g(3) = 5$, then $g(1) =$

(A) ~~−3.268~~
(B) −1.585
(C) ~~1.732~~
(D) 6.585
(E) 11.585

~~$g(x) = \int \sqrt{x^3+2}$ graph~~

$g(3) - g(1) = \int_1^3 f(x)$

$g(1) = g(3) - \int_1^3 f(x)$ use calc

Fund. Thm of calculus

$g(1) = 5 - \int_1^3 f(x) = \boxed{-1.585}$

22. Let g be the function given by $g(x) = \int_1^x 100\left(t^2 - 3t + 2\right)e^{-t^2}\,dt$.
Which of the following statements about g must be true?

I. g is increasing on (1, 2). $g'(x) = 100(x^2-3x+2)e^{-x^2}$ $g'(1)=0$ $g'(2)=0$

II. g is increasing on (2, 3).

~~III. $g(3) > 0$~~ $g(3) = \int_1^3 100\,(t^2-3t+2)e^{-t^2}\,dt = -1.942$

(A) I only
(B) II only
~~(C) III only~~
~~(D) II and III only~~
~~(E) I, II, and III~~

$g(1)=0$ $g(2)=-2.058$ slope $= \frac{-2.058-0}{2-1} = -2.058$ decreasing

$g(2)=-2.058$
$g(3)=-1.942$
$\frac{-1.942+2.058}{3-2}$ = positive

23. For a series S, let
$S = 1 - \frac{1}{9} + \frac{1}{2} - \frac{1}{25} + \frac{1}{4} - \frac{1}{49} + \frac{1}{8} - \frac{1}{81} + \frac{1}{16} - \frac{1}{121} + \cdots + a_n + \cdots,$

1.576 1.745

where $a_n = \begin{cases} \dfrac{1}{2^{(n-1)/2}} & \text{if } n \text{ is odd} \\ \dfrac{-1}{(n+1)^2} & \text{if } n \text{ is even.} \end{cases}$

$\frac{1}{2^n}$

$\frac{1}{(2n+1)^2}$ $\frac{1}{4n^2} \approx \frac{1}{n^2}$ p series ≤ 1 converges

Which of the following statements are true?

~~I. S converges because the terms of S alternate and $\lim_{n\to\infty} a_n = 0$.~~

II. S diverges because it is not true that $|a_{n+1}| < |a_n|$ for all n.

III. S converges although it is not true that $|a_{n+1}| < |a_n|$ for all n.

True

(A) None
~~(B) I only~~
(C) II only
(D) III only
~~(E) I and III only~~

Think as 2 different series

If two series converge then the whole [illegible] converges

ratio test $\frac{1}{2^{n+1}} \cdot 2^n = \frac{1}{2} < 1$ converges

ratio test $\frac{1}{(2(n+1)+1)^2} \cdot (2n+1)^2 = \frac{(2n+1)^2}{(2n+3)^2} =$

24. Let g be the function given by $g(t) = 100 + 20\sin\left(\frac{\pi t}{2}\right) + 10\cos\left(\frac{\pi t}{6}\right)$.
For $0 \le t \le 8$, g is decreasing most rapidly when $t =$

(A) 0.949 .6795
(B) 2.017 -35.94
(C) 3.106 -2.090
(D) 5.965 -31.44
(E) 8.000 35.95

graph
Find appropriate window

2nd CALC
look at $\frac{dy}{dx}$ at
given t.

FRQ

Name Answer key

A graphing calculator is required for some of these problems.

1. The table shows the depth of water, W, in a river, as measured at 4-hour intervals during a day-long flood. Assume that W is a differentiable function of time t.

t (hr)	0	4	8	12	16	20	24
$W(t)$ (ft)	32	36	38	37	35	33	32

(a) Find the approximate value of $W'(16)$. Indicate units of measure.

(b) Estimate the average depth of the water, in feet, over the time interval $0 \le t \le 24$ hours by using a trapezoidal approximation with subintervals of length $\Delta t = 4$ hours.

(c) Scientists studying the flooding believe they can model the depth of the water with the function $F(t) = 35 - 3\cos\left(\frac{t+3}{4}\right)$, where $F(t)$ represents the depth of the water, in feet, after t hours. Find $F'(16)$ and explain the meaning of your answer, with appropriate units, in terms of the river depth.

(d) Use the function F to find the average depth of the water, in feet, over the time interval $0 \le t \le 24$ hours.

+2

a) $W'(16) =$ slope at 16 (choose any two points around or including 16

(12, 37) and (16, 35) $\quad W'(16) = \frac{37-35}{12-16} = \frac{2}{-4} = \boxed{-\frac{1}{2}}$ ft/hr

+3

b) Trapezoidal $= \frac{\overset{\Delta t}{h}}{2}(y_0 + 2y_1 + 2y_2 + \dots + y_n)$

$= \frac{4}{2}(32 + 2(36) + 2(38) + 2(37) + 2(35) + 2(33) + 32)$

$= 2(422) = 844$ average depth over 24 hrs $= \frac{844}{24} = \boxed{35.167 \text{ ft}}$

+2

c) $F'(16) =$ nDeriv $\left(35 - 3\cos\left(\frac{x+3}{4}\right), x, 16\right)$

$= -.749$ since this is the slope at 16

it tells us that the depth is dropping at a rate of .749 ft/hr

+2

d) average depth = integrate

$\int_0^{24} 35 - 3\cos\left(\frac{x+3}{4}\right) = 842.779$ average ÷ 24

$= \boxed{35.116 \text{ ft}}$

No calculator

2. (a) Write the Maclaurin series (including the general term) for $f(x) = \ln(e + x)$.

(b) What is the radius of convergence?

(c) Use the first three terms of that series to write an expression that estimates the value of $\int_0^1 \ln(e + x^2)dx$.

+3

a) $f(0) = \ln(e+0) = \ln(e) = 1$

$\frac{f'(0)x}{1!}$ $\quad f'(x) = \frac{1}{e+x} = \frac{1}{e+0} = \frac{1}{e} \Rightarrow \frac{1}{e}\frac{x}{1!} = \frac{x}{e}$

$\frac{f''(0)x^2}{2!}$ $\quad f''(x) = \frac{-1}{(e+x)^2}$

$f''(0) = \frac{-1}{e^2} \Rightarrow \frac{-1}{e^2}\frac{x^2}{2!} = \frac{-x^2}{2e^2}$

$\frac{f'''(0)x^3}{3!}$ $\quad f'''(x) = \frac{2}{(e+x)^3} \Rightarrow \frac{\cancel{2}}{e^3} \cdot \frac{x^3}{3!} = \frac{x^3}{3e^3}$

$f'''(0) = \frac{2}{e^3}$ $\quad 3 \cdot \cancel{2} \cdot 1$

$$\boxed{1 + \frac{x}{e} - \frac{x^2}{2e^2} + \frac{x^3}{3e^3} + \dots + \frac{(-1)^{n+1}x^n}{ne^n}}$$

+3

b) $\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)e^{n+1}} \cdot \frac{ne^n}{x^n} \right| = \lim_{n\to\infty} \left| \frac{\cancel{x^n} \cdot x}{(n+1)\cancel{e^n} \cdot e^1} \cdot \frac{n \cdot \cancel{e^n}}{\cancel{x^n}} \right| = \lim_{n\to\infty} \frac{n}{en+e}|x|$

$\frac{1}{e}|x| < 1$

$|x| < e$

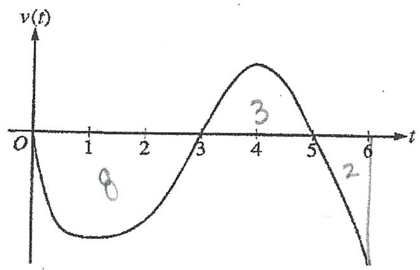
$\boxed{\text{radius of convergence is } e}$

+3

c) $\int_0^1 \ln(e+x^2)dx = \int_0^1 1 + \frac{x^2}{e} - \frac{(x^2)^2}{2e^2} = \int_0^1 1 + \frac{x^2}{e} - \frac{x^4}{2e^2}dx$

$= x + \frac{x^3}{3e} - \frac{x^5}{10e^2}\Big|_0^1 = \boxed{1 + \frac{1}{3e} - \frac{1}{10e^2}}$

Question 3



Graph of v

$t=0 \rightarrow x=-2$

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6$, is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the t-axis and the graph of v on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of t, where $0 \le t \le 6$, is the particle at $x = -8$? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

+3 a) since $v<0$ for $0<t<3$ and $5<t<6$
meaning the particle is moving left

$v>0$ for $3<t<5$ so the particle is moving right
lets look at the position at $t=3$ and $t=6$

below

$x(3) = -2 + \int_0^3 v(t)\,dt = -2 + (-8) = \boxed{-10}$

$x(6) = -2 + \int_0^6 v(t)\,dt = -2 + (-8) + 3 + (-2) = -10 + 1 = -9$

$\boxed{\text{at } t=3 \text{ and pocition } -10}$

+3 b) position at time t

$t=0$ position $= -2$ > moving L

$t=3$ positon $= -10$ > moving R

$x(5) = -2 + \int_0^5 v(t)\,dt = -2 + (-8) + 3 = -10 + 3$ $t=5$ position $= -7$ > moving L

$t=6$ pocition $= -9$

$\boxed{\text{3 times}}$

Question # 3

+1

c) on the interval $2<t<3$

velocity is negative $v<0$

concave up → acceleration is positive $a>0$

Since the velocity and acceleration are different signs

$\boxed{\text{The speed is decreasing}}$

+2

d) when acceleration is negative is when the slope of the velocity graph is decreasing (negative slope)

$\boxed{0<t<1 \text{ and } 4<t<6}$