Calculus I – Worksheet #19
Review for Test 4 - Derivatives

1. Find the derivative of $y = x^2$ with respect to $\ln x$.	2. Find the derivative of y^2 with respect to x^4 if $y = \sqrt{x^2 + 5}$.
3. Simplify: $e^{4 \ln x}$	4. Find $\frac{dy}{dx}$ if $y = 3^{x+1}$
5. Find $\frac{dy}{dx}$ if $y = (\ln x)^x$	6. Find $\frac{dy}{dx}$ if $y = \sqrt{4x^2 + 4x}$
7. Find $\frac{dy}{dx}$ if $y = \cos x \sin x$	8. Find $\frac{dy}{dx}$ if $y = \cos^3 5x$
9. Find $\frac{dy}{dx}$ if $y = \ln(xe^{2x})$	10. Which of the following functions does not have a derivative equal to $\frac{1}{x}$? A. $\ln(ex)$ B. $\ln(2x)$ C. $\ln(e^{\ln x})$ D. $\ln(xe^x)$
11. Write the equation of the tangent line to $f(x) = e^{2x}$ at x = 2.	12. Find $\frac{dy}{dx}$ if $y = \ln(6x^2 - 3)$
13. Simplify: $e^{4x+2\ln x}$	14. Find $\frac{dy}{dx}$ if $y = \ln\lvert\sin 3x\rvert$.
15. Find $\frac{dy}{dx}$ if $y = 2^{\cos x}$	16. Find $\frac{dy}{dx}$ if $y = \ln\left(\frac{x^2}{e^{6x}}\right)$
17. Find $\frac{dy}{dx}$ if $y = x^2 \sec 4x$	18. Find the derivative of $y = x^3 + 2x^2$ with respect to $\cos x$.
19. Find $\frac{dy}{dx}$ if $y = x^{\cot 2x}$.	20. Find $\frac{dy}{dx}$ if $y = \cos^2(3x) + \sin^2(3x)$

Answers:

1. $2x^2$	2. $\frac{1}{2x^2}$	3. x^4	4. $3^{x+1} \ln 3$
5. $(\ln x)^x \left[\frac{1}{\ln x} + \ln(\ln x)\right]$	6. $\frac{2x+1}{\sqrt{x^2+x}}$	7. $\cos 2x$	8. $-15\cos^2 5x \sin 5x$
9. $\frac{1+2x}{x}$	10. D	11. $y - e^4 = 2e^4(x-2)$	12. $\frac{12x}{6x^2-3} = \frac{4x}{2x^2-1}$
13. $x^2 e^{4x}$	14. $3\cot 3x$	15. $-2^{\cos x}(\ln 2)(\sin x)$	16. $\frac{2-6x}{x}$
17. $2x \sec 4x(1 + 2x \tan 4x)$	18. $\frac{-3x^2-4x}{\sin x}$	19. $\left(-2\ln x \csc^2 2x + \frac{\cot 2x}{x}\right) x^{\cot x}$	20. 0

① $y = x^2$ $\quad y' = \dfrac{2x}{\frac{1}{x}} = 2x \cdot \dfrac{x}{1} = \boxed{2x^2}$

② $y = \sqrt{x^2+5}$

$y^2 = x^2+5 \Rightarrow y' = \dfrac{2x}{4x^3} = \boxed{\dfrac{1}{2x^2}}$

③ $e^{4\ln x} = e^{\ln x^4} = \boxed{x^4}$

④ $y = 3^{x+1}$ $\quad y' = \ln(3) \cdot 3^{x+1} \cdot 1 = \boxed{3^{x+1}\ln(3)}$

⑤ $y = (\ln x)^x$

$\ln y = \ln(\ln x)^x$

$\ln y = x \cdot \ln(\ln x)$

⑥ $y = \sqrt{4x^2+4x}$ $\quad y' = \frac{1}{2}(4x^2+4x)^{-1/2} \cdot 8x+4$

$= \dfrac{4x+2}{\sqrt{4x^2+4x}} = \dfrac{4x+2}{\sqrt{4(x^2+x)}} = \dfrac{4x+2}{2\sqrt{x^2+x}}$

$fg' + gf'$

$\dfrac{1}{y} = x \cdot \dfrac{1}{\ln x} \cdot \dfrac{1}{x} + \ln(\ln x) \cdot 1$

$y \cdot \dfrac{1}{y}\dfrac{dy}{dx} = \left[\dfrac{1}{\ln x} + \ln(\ln x)\right] \cdot y$

$= \boxed{\dfrac{2x+1}{\sqrt{x^2+x}}}$

$\boxed{(\ln x)^x \left[\dfrac{1}{\ln x} + \ln(\ln x)\right]}$

⑦ $y = \cos x \cdot \sin x$ $\quad fg' + gf'$

$y' = \cos x \cdot \cos x + \sin x \cdot -\sin x$

$y' = \cos^2 x - \sin^2 x$

$= \boxed{\cos(2x)}$

$\cos^2 x - \sin^2 x = \cos 2x$

⑧ $y = \cos^3(5x) \Rightarrow$ $\quad y' = 3\cos^2(5x) \cdot -\sin(5x) \cdot 5$

$\boxed{y' = -15\cos^2(5x)\sin(5x)}$

⑨ $y = \ln(x \cdot e^{2x})$ $\quad fg' + gf'$

$y' = \frac{1}{xe^{2x}} \cdot [x \cdot e^{2x} \cdot 2 + e^{2x} \cdot 1]$

$= \frac{2xe^{2x} + e^{2x}}{xe^{2x}} = \frac{e^{2x}(2x+1)}{xe^{2x}} = \boxed{\frac{2x+1}{x}}$

⑩ A) $\ln(ex) \Rightarrow \frac{1}{ex} \cdot [e] = \frac{1}{x}$ ✓

B) $\ln(2x) \Rightarrow \frac{1}{2x} \cdot 2 = \frac{1}{x}$ ✓

C) $\ln(e^{\ln x}) = \ln(x) \Rightarrow \frac{1}{x}$ ✓

D) $\ln(xe^x) = \frac{1}{xe^x} \cdot [x \cdot e^x + e^x \cdot 1]$ $\quad \frac{xe^x + e^x}{xe^x} = \frac{x+1}{x}$

(11) · skip for now (4

(12) $y = \ln(6x^2 - 3)$

$$y' = \frac{1}{6x^2-3} \cdot 12x = \frac{12x}{3(2x^2-1)} = \boxed{\frac{4x}{2x^2-1}}$$

(13) $e^{4x+2\ln x} = e^{4x} \cdot e^{2\ln x}$

$$= e^{4x} \cdot e^{\ln x^2}$$

$$= e^{4x} \cdot x^2 = \boxed{x^2 \cdot e^{4x}}$$

(14) $y = \ln|\sin 3x|$

$$y' = \frac{1}{\sin 3x} \cdot \cos(3x) \cdot 3$$

$$= 3\frac{\cos(3x)}{\sin(3x)} = \boxed{3\cot(3x)}$$

(15) $y = 2^{\cos x}$

$$y' = \ln(2) \cdot 2^{\cos x} \cdot -\sin x$$

$$= \boxed{-2^{\cos x}(\ln 2)(\sin x)}$$

(16) $y = \ln\left(\frac{x^2}{e^{6x}}\right)$ $\quad \frac{gf' - fg'}{g^2}$

$$y' = \frac{1}{\frac{x^2}{e^{6x}}}\left[\frac{e^{6x}\cdot 2x - x^2\cdot e^{6x}\cdot 6}{(e^{6x})^2}\right.$$

$$\frac{e^{6x}}{x^2}\left[\frac{e^{6x}(2x-6x^2)}{(e^{6x})^2}\right. = \frac{2x-6x^2}{x^2}$$

$$= \boxed{\frac{2-6x}{x}}$$

(17) $y = x^2 \sec(4x)$ $\quad fg' + gf'$

$$y' = x^2\cdot \sec(4x)\tan(4x)\cdot 4 + \sec(4x)\cdot 2x$$

$$4x^2\cdot\sec(4x)\tan(4x) + 2x\sec(4x)$$

$$\boxed{2x\cdot\sec(4x)\left[2x\tan(4x)+1\right]}$$

(19) ~~$y = x\cos(2x)$~~

(20) $y = \cos^2(3x) + \sin^2(3x)$

$$y' = 2\cos(3x)\cdot -\sin(3x)\cdot 3 + 2\sin(3x)\cdot\cos(3x)\cdot 3$$

$$= -6\cos(3x)\sin(3x) + 6\cos(3x)\sin(3x)$$

$$= \boxed{0}$$