Answer key (!!

Part B TIME: 50 MINUTES

Some questions in this part of the examination require the use of a graphing calculator. There are 17 questions in Part B, for which 50 minutes are allowed. Because there is no deduction for wrong answers, you should answer every question, even if you need to guess.

Directions: Choose the best answer for each question. If the exact numerical value of the correct answer is not listed as a choice, select the choice that is closest to the exact numerical answer.

The path of a satellite is given by the parametric equations

$$x = 4\cos t + \cos 12t,$$

$$y = 4\sin t + \sin 12t.$$

velocity y' and $t=1$

1. The upward velocity at $t = 1$ equals

(A) 2.829 (B) 3.005 (C) 3.073 (D) 3.999 (E) 12.287

Initial temp = 120° then $t=0$

Find K

$120 = 70 + Ke^{-0.4(0)}$

$120 = 70 + Ke^{0}$

$120 = 70 + K$

$50 = K$

2. As a cup of hot chocolate cools, its temperature after t minutes is given by $H(t) = 70 + ke^{-0.4t}$. If its initial temperature was 120°F, what was its average temperature (in °F) during the first 10 minutes?

(A) 60.9 (B) 82.3 (C) 95.5 (D) 96.1 (E) 99.5

$\frac{1}{10}\int_0^{10} 70 + 50e^{-0.4t}$

$u = \sqrt{x-2}$ $u^2 = x-2$ $u^2+2 = x$

3. If $\sqrt{x-2}$ is replaced by u, then $\int_3^6 \frac{\sqrt{x-2}}{x}\,dx$ is equivalent to

$\int_1^2 \frac{u}{u^2+2} \cdot 2u\,du$

(A) $\int_1^2 \frac{u\,du}{u^2+2}$ (B) $2\int_1^2 \frac{u^2\,du}{u^2+2}$ (C) $\int_3^6 \frac{2u^2}{u^2+2}\,du$

$u = \sqrt{6-2} = \sqrt{4} = 2$

(D) $\int_3^6 \frac{u\,du}{u^2+2}$ (E) $\int_1^2 \frac{u^2}{u^2+2}\,du$

$u = \sqrt{3-2} = \sqrt{1} = 1$

$x = 3$ $\frac{3^n}{(n+1)3^n} = \frac{1}{n+1}$ Diverges

$-3 \le x < 3$

4. The set of all x for which the power series $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)\cdot 3^n}$ converges is

$x = -3$ $\frac{(-3)^n}{(n+1)3^n} = \pm 1$ - alt. series converge

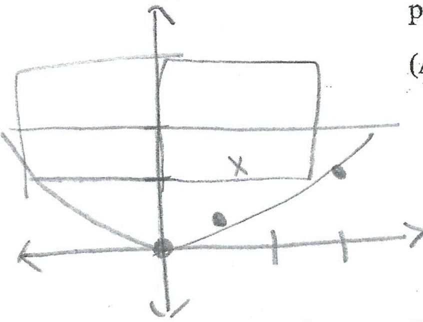
(A) $\{-3, 3\}$ (B) $|x| < 3$ (C) $|x| > 3$
(D) $-3 \leqq x < 3$ (E) $-3 < x \leqq 3$

$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+2)3^{n+1}} \cdot \frac{3^n(n+1)}{x^n} \right| = \lim_{n\to\infty} \left| \frac{\cancel{x^n}\, x}{(n+2)\cancel{3^n}\, 3^1} \cdot \frac{\cancel{3^n}(n+1)}{\cancel{x^n}} \right|$

$= \lim_{n\to\infty} \left| \frac{n+1}{3n+6} \cdot x \right| = \frac{1}{3}|x| < 1$

$-3 < |x| < 3$

$\int_0^2 16y\,dy$



5. The base of a solid is the region bounded by $x^2 = 4y$ and the line $y = 2$, and each plane section perpendicular to the y-axis is a square. The volume of the solid is

(A) 8 (B) 16 (C) 20 (D) 32 (E) 64

$(2x)^2$

$4x^2 = 16y$

$1 = 4y$

$2^2 = 4y$

$\int_0^2 s^2\,dy$

$\int_0^2 4y\,dy$

6. A cup of coffee placed on a table cools at a rate of $\frac{dH}{dt} = -0.05(H-70)$°F per minute, where H represents the temperature of the coffee and t is time in minutes. If the coffee was at 120°F initially, what will its temperature be 10 minutes later?

(A) 73°F (B) 95°F (C) 100°F (D) 118°F (E) 143°F

$\ln|H-70| = -0.05t + \ln 50$

$\ln|H-70| = -0.05(10) + \ln 50$

$\ln|H-70| = -.5 + \ln 50$

$H - 70 = e^{-.5} \cdot \cancel{e^{\ln 50}}$

$H - 70 = 50e^{-.5}$

$+70 \quad +70$

$H = 100.327$

$\frac{dH}{H-70} = -0.05\,dt$

$\ln|H-70| = -0.05t + C$

$(0, 120)$

$\ln|120-70| = -0.05(0) + C$

$\ln|50| = C$

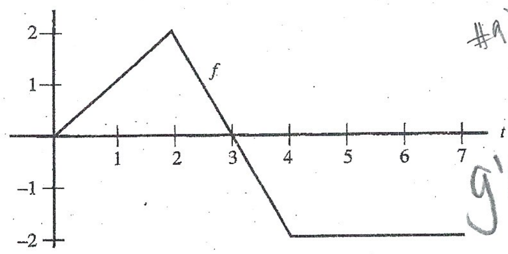
7. Find the area bounded by the spiral $r = \ln\theta$ on the interval $\pi \leq \theta \leq 2\pi$.

(A) 2.405 (B) 2.931 (C) 3.743 (D) 4.810 (E) 7.487

$r = 0$

$A = \int \frac{1}{2} r^2\,d\theta = \int_{\pi}^{2\pi} \frac{1}{2}(\ln\theta)^2\,d\theta =$

uestions 8 and 9: Use the graph of f shown on [0,7]. Let $G(x) = \int_2^{3x-1} f(t)\,dt$.



$g'(x) = 3f(3x-1)$

$g'(1) = 3f(3(1)-1)$

$= 3f(3-1)$

$= 3f(2) = 3 \cdot 2 = 6$

#9) $f(3x-1) \cdot 3$

$g'(x) = 3f(3x-1) = 0$

$f(3x-1) = 0$

$f = 0$ at $t = 3$

8. $G'(1)$ is

(A) 1 (B) 2 (C) 3 (D) 6 (E) undefined

9. G has a local maximum at $x =$ when does $G'(x) = 0$

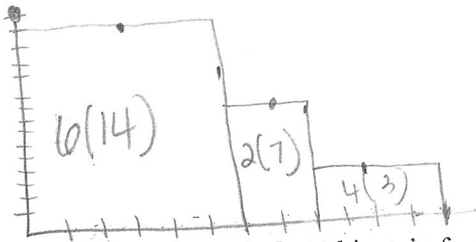
(A) 1 (B) $\frac{4}{3}$ (C) 2 (D) 5 (E) 8

$3x - 1 = 3$

$+1 \quad +1$

$\frac{3x}{3} = \frac{4}{3}$

$x = \frac{4}{3}$



$6(14)+2(8)+4(3)$
$84+16+12$

10. The table shows the speed of an object, in feet per second, at various times during a 12-second interval.

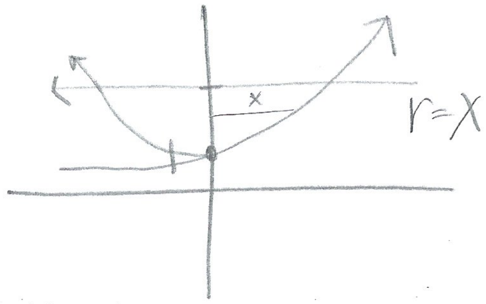
time (sec)	0	3	6	7	8	10	12
speed (ft/sec)	15	14	11	8	7	3	0

Estimate the distance the object travels, using the midpoint method with 3 subintervals.

(A) 100 ft (B) 101 ft (C) 111 ft (D) 112 ft (E) 150 ft

$$\int \pi r^2 = \int_1^2 \pi x^2\,dy$$
$$= \int_1^2 \pi(\ln y)^2\,dy$$
$$= 0.592$$

$\ln y$ ~~the~~ x
$\ln y = x$



11. Find the volume of the solid generated when the region bounded by the y-axis, $y=e^x$, and $y=2$ is rotated around the y-axis.

(A) 0.296 (B) 0.592 (C) 2.427 (D) 3.998 (E) 27.577

12. If $f(t)=\int_0^{t^2} \frac{1}{1+x^2}\,dx$, then $f'(t)$ equals

(A) $\frac{1}{1+t^2}$ (B) $\frac{2t}{1+t^2}$ (C) $\frac{1}{1+t^4}$ (D) $\frac{2t}{1+t^4}$ (E) $\tan^{-1} t^2$

$$\frac{1}{1+(t^2)^2}\cdot 2t = \frac{2t}{1+t^4}$$

must find $\frac{da}{dt}$

$a^2+b^2=c^2$

$2a\cdot\frac{da}{dt}+2b\cdot\frac{db}{dt}=2c\frac{dc}{dt}$ $\qquad \frac{da}{dt}=\frac{12}{8}=\frac{3}{2}$

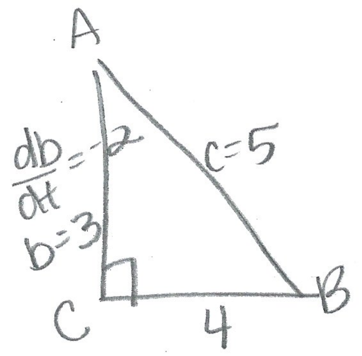
$2\cdot4\cdot\frac{da}{dt}+2\cdot3\cdot-2=2\cdot5\cdot0$

$8\frac{da}{dt}-12=0$

alt. terms and even power + factorial

13. For which function is $\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$ the Taylor series about 0?

(A) ~~e^x~~ (B) ~~e^{-x}~~ (C) ~~$\sin x$~~ (D) $\cos x$ (E) $\ln(1+x)$



14. The hypotenuse AB of a right triangle ABC is 5 feet, and one leg, AC, is decreasing at the rate of 2 feet per second. The rate, in square feet per second, at which the area is changing when $AC = 3$ is

(A) $\frac{25}{4}$ (B) $\frac{7}{4}$ (C) $-\frac{3}{2}$ (D) $-\frac{7}{4}$ (E) $-\frac{7}{2}$

$A=\frac{1}{2}\overset{f}{b}\cdot\overset{g}{a}$

$fg'+gf'$

$\frac{dA}{dt}=\frac{1}{2}\left[b\cdot\frac{da}{dt}+a\cdot\frac{db}{dt}\right]$

$=\frac{1}{2}\left[3\cdot\frac{3}{2}+4\cdot-2\right]$

$\frac{1}{2}\left[\frac{9}{2}-\frac{8\cdot2}{1\cdot2}\right]$

$\frac{1}{2}\left[\frac{9}{2}-\frac{16}{2}\right]$

$\frac{1}{2}\left[\frac{-7}{2}\right]=\frac{-7}{4}$

15. At how many points on the interval $[0,\pi]$ does $f(x) = 2\sin x + \sin 4x$ satisfy the Mean Value Theorem?

(A) none (B) 1 (C) 2 (D) 3 (E) 4

16. Which one of the following series converges?

(A) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ $p=\frac{1}{2}$ (B) $\sum_{n=1}^{\infty}\frac{1}{n}$ $p=1$ (C) $\sum_{n=1}^{\infty}\frac{1}{2n+1}$ n $p=1$

(D) $\sum_{n=1}^{\infty}\frac{\cancel{n}}{\cancel{n}^2+1}$ $\frac{1}{n}$ $p=1$ (E) $\sum_{n=1}^{\infty}\frac{1}{n^2+1}$ $p=2$

17. The rate at which a purification process can remove contaminants from a tank of water is proportional to the amount of contaminant remaining. If 20% of the contaminant can be removed during the first minute of the process and 98% must be removed to make the water safe, approximately how long will the decontamination process take?

(A) 2 min (B) 5 min (C) 18 min (D) 20 min (E) 40 min

$0.02=0.8^t$

solve $t=17.53$

STOP

END OF SECTION I

#15)

$f(0)=2\sin(0)+\sin(4\cdot0)$

$=0+0$

$f(0)=0$

$f(\pi)=2(\sin\pi)+\sin(4\pi)$

0 0

slope $\frac{0-0}{\pi-0}=0$

how many times is the slope zero between 0 and π

look at graph = 4