Answer key (4)

Calculus 1 Worksheet #16
Derivatives of Trigonometric Functions

Notes: Know the following theorems.

1. $\frac{d(\tan\square)}{dx} = \sec^2\square \cdot \frac{d\square}{dx}$	2. $\frac{d(\cot\square)}{dx} = -\csc^2\square \cdot \frac{d\square}{dx}$	3. $\frac{d(\sec\square)}{dx} = \sec\square \cdot \tan\square \cdot \frac{d\square}{dx}$	4. $\frac{d(\csc\square)}{dx} = -\csc\square \cdot \cot\square \cdot \frac{d\square}{dx}$

Examples:

1. $y = \tan 5x$ $y' = 5\sec^2 5x$	2. $y = \sec 5x$ $y' = 5\tan 5x \sec 5x$	3. $y = \cot^4 3x$ $y' = 4\left[-\cot^3 3x \csc^2 3x\right](3)$ $y' = -12\cot^3 3x \csc^2 3x$	4. $y = \csc^3 2x$ $y' = 3(\csc^2 2x)[-\csc 2x \cot 2x](2)$ $y' = \boxed{-6(\csc^2 2x)\csc 3x \cot 3x}$

Use the **quotient rule** to **prove** the derivative of: [Hint: change into sin x and cos x and then take derivative]
1. tan x 2. cot x 3. sec x 4. csc x

Directions: Find dy/dx.

5. y = sec4x	6. y = tan3x – cot3x	7. y = cot5x + csc5x
8. $y = \csc^3(2x)$	9. y = tanx + cotx	10. y = 4secx – 2cscx
11. $y = 3\sec x(\tan x)$	12. $y = \sin x(\tan x)$	13. $y = \cot x(\csc x)$
14. $y = \cos x(\cot x)$	15. $y = \frac{2\cos x}{x+1}$	16. $y = \frac{\sin x}{x}$
17. $y = \frac{\sin x}{1-\cos x}$	18. $y = \frac{x+2}{\cos x}$	19. $y = \frac{\tan x}{\cos x - 4}$
20. $y = \frac{\cot x}{1-\sin x}$		

Answers:

1. $\sec^2 x$	2. $-\csc^2 x$	3. $\sec x \tan x$
4. $-\csc x \cot x$	5. $4\sec 4x \tan 4x$	6. $3(\sec^2 3x + \csc^2 3x)$
7. $-5\csc 5x(\csc 5x + \cot 5x)$	8. $-6\csc^3(2x)\cot(2x)$	9. $\sec^2 x - \csc^2 x$
10. $2(2\sec x \tan x + \csc x \cot x)$	11. $3\sec x(\tan^2 x + \sec^2 x)$	12. $\sin x(1+\sec^2 x)$
13. $-\csc x(\csc^2 x + \cot^2 x)$	14. $-\cos x(1+\csc^2 x)$	15. $\frac{-2(x\sin x + \sin x + \cos x)}{(x+1)^2}$
16. $\frac{x\cos x - \sin x}{x^2}$	17. $\frac{1}{\cos x - 1}$	18. $\frac{\cos x + x\sin x + 2\sin x}{\cos^2 x}$
19. $\frac{\sec x - 4\sec^2 x + \tan x \sin x}{(\cos x - 4)^2}$	20. $\frac{-\csc^2 x + \csc x + \cot x \cos x}{(1-\sin x)^2}$	

① $\tan x = \frac{\sin x}{\cos x}$ $\quad \frac{gf' - fg'}{g^2}$

$$y' = \frac{\cos x \cdot \cos x - \sin x \cdot -\sin x}{\cos^2 x} = \frac{\cos^2 + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$= \boxed{\sec^2 x}$$

② $\cot x = \frac{\cos x}{\sin x}$ $\quad \frac{gf' - fg'}{g^2}$

$$y' = \frac{\sin x \cdot -\sin x - \cos x \cdot \cos x}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x}$$

$$= \boxed{-\csc^2 x}$$

③ $\sec x = \frac{1}{\cos x}$ $\quad \frac{gf' - fg'}{g^2}$

$$y' = \frac{\cos x \cdot 0 - 1 \cdot -\sin x}{\cos^2 x} = \frac{+\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

$$= \boxed{\tan x \cdot \sec x}$$

④ $\csc x = \frac{1}{\sin x}$ $\frac{gf' - fg'}{g^2}$

$$y' = \frac{\sin x \cdot 0 - 1 \cdot \cos x}{\sin^2 x} = \frac{-\cos x}{\sin^2 x} = \frac{-\cos x}{\sin x} \cdot \frac{1}{\sin x}$$

$$= \boxed{-\cot x \cdot \csc x}$$

⑤ $y = \sec 4x$

$y' = \sec 4x \cdot \tan 4x \cdot 4$ $= \boxed{4 \sec 4x \cdot \tan 4x}$

⑥ $y = \tan 3x - \cot 3x$

$y' = \sec^2 3x \cdot 3 + + \csc^2 3x \cdot 3$

$= \boxed{3\sec^2(3x) + 3\csc^2(3x)}$

⑦ $y = \cot(5x) + \csc(5x)$

$y' = -\csc^2(5x) \cdot 5 + -\csc(5x)\cot(5x) \cdot 5$

$y' = -5\csc^2(5x) - 5\csc(5x)\cot(5x)$

$\boxed{-5\csc(5x)\left(\csc(5x) + \cot(5x)\right)}$

(8) $y = \csc^3(2x)$

$y' = 3\csc^2(2x) \cdot -\csc(2x)\cot(2x) \cdot 2$

$= \boxed{-6\csc^3(2x)\cot(2x)}$

(9) $y = \tan x + \cot x$

$y' = \sec^2 x + -\csc^2 x$

$\boxed{\sec^2 x - \csc^2 x}$

(10) $y = 4\sec x - 2\csc x$

$y' = 4\sec x \cdot \tan x + {}^{+}2\csc x \cdot \cot$

$\boxed{4\sec x \tan x + 2\csc x \cot x}$

(11) $y = 3\sec^{f} x \cdot (^{g}\tan x)$ $\quad fg + gf'$

$y' = 3[\sec x \cdot \sec^2 x + \tan x \cdot \sec x \tan x)$

$= 3[\sec^3 x + \tan^2 x \cdot \sec x]$

$= \boxed{3\sec x [\sec^2 x + \tan^2 x]}$

(12) $y = \sin x \cdot (\tan x)$ $fg' + gf'$

$y' = \sin x \cdot \sec^2 x + \tan x \cdot \cos x$

$= \sin x \cdot \frac{1}{\cos^2 x} + \frac{\sin x}{\cos x} \cdot \cos x$

$= \sin x \cdot \sec^2 x + \sin x$

$= \boxed{\sin x (\sec^2 x + 1)}$

(13) $y = \cot x \cdot (\csc x)$ $fg' + gf'$

$y = \cot x \cdot -\csc x \cdot \cot x + \csc x \cdot -\csc^2 x$

$y = -\cot^2 x \csc x - \csc^3 x$

$\boxed{y = -\csc x (\cot^2 x + \csc^2 x)}$

(14) $y = \cos x \cdot \cot x$ $fg' + gf'$

$y' = \cos x \cdot -\csc^2 x + \cot x \cdot -\sin x$

$y' = -\cos x \cdot \frac{1}{\sin^2 x} - \frac{\cos x}{\sin x} \cdot \sin x$

$y' = -\frac{\cos x}{} \cdot \csc^2 x - \cos x$ $\boxed{-\cos x (\csc^2 x + 1)}$

(15) $y = \frac{2\cos x}{x+1}$ $\qquad \frac{gf' - fg'}{g^2}$

$$y' = \frac{(x+1)(-2\sin x) - (2\cos x)(1)}{(x+1)^2}$$

$$= \frac{-2x\sin x - 2\sin x - 2\cos x}{(x+1)^2}$$

$$= \boxed{\frac{-2(x\sin x + \sin x + \cos x)}{(x+1)^2}}$$

(16) $y = \frac{\sin x}{x}$ $\qquad \frac{gf' - fg'}{g^2}$

$$y' = \frac{x \cdot \cos x - \sin x \cdot 1}{x^2}$$

$$\boxed{y' = \frac{x \cdot \cos x - \sin x}{x^2}}$$

(17) $y = \frac{\sin x}{1 - \cos x}$ $\qquad \frac{gf' - fg'}{g^2}$

$$y' = \frac{(1 - \cos x) \cdot \cos x - \sin x \cdot (+ + \sin x)}{(1 - \cos x)^2}$$

$$= \frac{\cos x - \cos^2 x - \sin^2 x}{(1 - \cos x)^2} \qquad -1$$

$$= \frac{\cos x - 1}{(1 - \cos x)^2} = \frac{-(1 - \cos x)}{(1 - \cos x)^2} = \frac{-1}{1 - \cos x} \qquad \div -1 \quad \div -1$$

$$\boxed{\frac{1}{\cos x - 1}}$$

#18 $y = \frac{x+2}{\cos x}$ $\qquad \frac{gf' - fg'}{g^2}$

$$y' = \frac{\cos x(1) + (x+2)(+\sin x)}{\cos^2 x}$$

$$= \boxed{\frac{\cos x + x\sin x + 2\sin x}{\cos^2 x}}$$

#19 $y = \frac{\tan x}{\cos x - 4}$ $\qquad \frac{gf' - fg'}{g^2}$

$$y' = \frac{(\cos x - 4)\cdot(\sec^2 x) + \tan x(+\sin x)}{(\cos x - 4)^2}$$

($\sec^2 x = \frac{1}{\cos^2 x}$)

$$= \boxed{\frac{\sec x - 4\sec^2 x + \tan x \cdot \sin x}{(\cos x - 4)^2}}$$

(20) $y = \dfrac{\cot x}{1-\sin x}$ $\qquad \dfrac{gf' - fg'}{g^2}$

$\frac{1}{\sin^2 x}$

$$y' = \frac{(1-\sin x)\cdot(-\csc^2 x) - \cot x(-\cos x)}{(1-\sin x)^2}$$

$$\boxed{\frac{-\csc^2 x + \csc x + \cot x \cdot \cos x}{(1-\sin x)^2}}$$